63-2-3

AIR FORCE INSTITUTE OF TECHNOLOGY

AIR UNIVERSITY

UNITED STATES AIR FORCE

SCHOOL OF ENGINEERING

WRIGHT–PATTERSON AIR FORCE BASE, OHIO

AF-WP-O-MAY 62 3,500

THESIS

Presented to the Faculty of the School of Engineering

The Air Force Institute of Technology

Air University

in Partial Fulfillment of the

Requirements for the

Master of Science Degree

in Electrical Engineering

A COMPUTER MODEL FOR THE DETERMINATION OF FOUR-LEVEL LASER EFFICIENCY

by

Tom Gordon Purnhagen, B.E.E.

Captain, USAF GE/EE/62-18

Graduate Electronics

December 1962

AF-WP-O-JAN 63 20

Preface

The mathematical model described in this paper is intended primarily for use by systems engineers, and for that reason I have adopted to a degree the viewpoint of the systems engineer rather than that of the theoretical physicist in its development. I have made a determined effort to describe the four-level Laser system in terms of parameters which are physically measurable in the laboratory by means of relatively simple and straightforward methods. Whether or not such an approach is adequate to the problem at hand remains to be determined through the actual application of the model to physical systems.

I wish to express my gratitude to the many individuals who have assisted me in this study. In particular, I wish to thank Mr. Sirons, of the Navigation and Guidance Laboratory, Aeronautical Systems Division, who sponsored the project and supplied the equipment for the study of xenon lamp pulses and the sample for spectroscopic analysis, Mr. Blasingame, of the Electronic Technology Laboratory, ASD, who performed the spectroscopy, and Professor Lubelfeld, of the Electrical Engineering Department, Air Force Institute of Technology, my Faculty Thesis Adviser.

Tom G. Purnhagen

Contents

List of Figures

List of Tables

Abstract

The Air Force is considering several pulsed four-level Lasers for possible use in weapons systems. So far, the advantages of these Lasers over the ruby Laser, such as lower threshold and reduced spiking, have been offset by their lower efficiency. Although four-level Laser efficiency could be maximized by experimental means, analysis by means of a mathematical model appears more suitable for economic reasons. Such a model is developed in this paper by identifying the loss mechanisms which operate in the four-level Laser system, defining the system efficiency as the product of six sub-efficiencies, one associated with each loss mechanism, and deriving equations for the sub-efficiencies in terms of physically measurable parameters of the Laser system. The model is limited in its present form to the representation of pulsed four-level Lasers employing helical or annular lamp pumping, but could be easily extended to cover other pumping geometries and continuous operation.

Based on the mathematical model, a program is written for the IBM 1620 digital computer which calculates the efficiency of a pulsed four-level Laser oscillator or the maximum efficiency and maximum gain of a pulsed four-level

Laser amplifier. The program is written in a modification of IBM FORTRAN, and consists of three sub-programs; one which need only be run once, one which must be run once for each Laser material, and one which produces the final results. Output data from part 1 are listed, and detailed operating instructions for parts 2 and 3 are given. A sample problem involving the design of a neodymium doped glass Laser oscillator is solved to illustrate the use of the model. On the basis of the model and the experience gained in its development and testing, it can be concluded that the efficiency of pulsed four-level Lasers will tend to increase as length, radius, doping density, and pumping energy are increased, and decrease as end reflectivities and pumping pulse time constant are increased, within the limits imposed by the assumptions and approximations used in the development of the model.

The results produced by the model have not been checked experimentally, and it is recommended that such a check be made before the model is used in actual design problems.

A COMPUTER MODEL FOR THE DETERMINATION OF FOUR-LEVEL LASER EFFICIENCY

I. Introduction

The United States Air Force is presently conducting research to determine the potentialities of the Laser as a military weapon. Although much of the current effort is centered about the ruby Laser, which was the first solid state Laser developed and hence is the most highly perfected, the interest of the Air Force is by no means limited to that particular device. Numerous other materials are also being investigated, among which are various crystals, glasses, and liquids doped with divalent or trivalent rare earth ions.

The rare earth Lasers operate in a four-level energy configuration (Figure 1b) as opposed to the three-level ruby Laser (Figure 1a). A comparison of the two energy level configurations indicates that the four-level system has some inherent advantages over the three-level system, among which are a lower threshold and reduced spiking (relaxation oscillations). For high power applications, however, these advantages have been offset by the relative inefficiency of existing four-level systems compared with

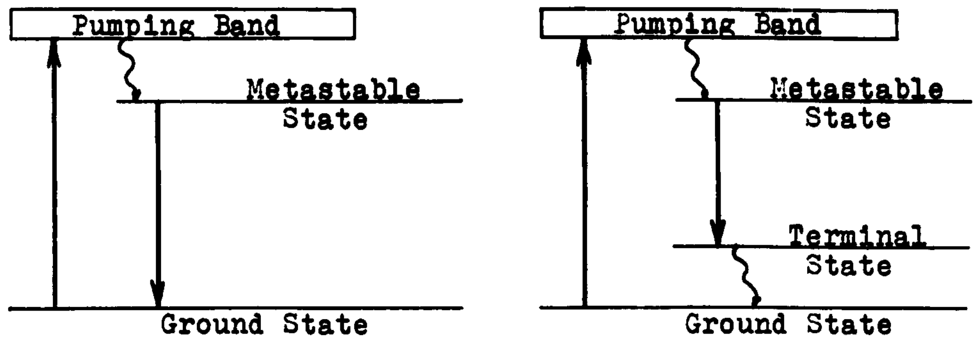

Fig. 1

(a) Three-level System. (b) Four-level System.

the ruby. If the rare earth Lasers are to be useful in weapons systems, then, it is important to be able to design and operate them in the most efficient manner possible.

The problem of maximizing Laser efficiency can be (and has been) approached through experimental study; that is, through trying various combinations of physical parameters, doping levels, end mirror reflectivities, and so forth, to see which combination gives the best results. This method, although practical, is expensive and time consuming. A better approach would seem to be to develop a mathematical model for the Laser system, and to analytically (or graphically) determine the optimum design prior to the actual physical construction of the Laser.

The remainder of this paper is devoted to the development of a program for the IBM 1620 digital computer which calculates the efficiency of a pulsed four-level Laser from the values of the system parameters and certain physically measurable characteristics of the Laser material and the pumping source. Although the program and the mathematical model from which it is derived were originally designed to represent Lasers employing trivalent neodymium, it is believed that the results are applicable directly to a broad class of four-level Lasers, and that only minor changes would be required to further extend the range of applicability.

Chapter II of this paper identifies the loss mechanisms in the four-level Laser system and defines the overall efficiency in terms of six sub-efficiencies. Chapter III lists the more important assumptions and approximations on which the analysis is based, and indicates how these will affect the range of applicability of the model. The actual development of the mathematical model is presented in Chapter IV, and in Chapter V this model is translated into the computer program. Chapter VI illustrates the application of the computer program to a design problem, and Chapter VII is a summary of the results and conclusions which may be drawn from this study.

II. The Loss Mechanisms

Before a model can be developed to determine the efficiency of the four-level Laser system, it is necessary to identify the loss mechanisms which are operative in the system. This can be done by following the energy through the system from input to output and noting the losses which occur.

Lamp Losses

Nearly all the Laser systems now in use employ some sort of transducer which converts electrical energy into photon energy to excite the active ions in the Laser itself. In all such transducers there is some loss of energy due to the inefficiency of the conversion process. Accordingly, a quantity η_L, the lamp efficiency, can be defined as the ratio of the output energy of a given pumping source which lies within a specified range of wavelength to the total input energy to that source.

Geometrical Losses

The portion of the photon energy produced by the pumping source which actually enters the Laser rod will depend on the geometry of the lamp and enclosure, and to some degree on other factors such as the refractive index of the Laser material and the reflectivity of the walls of the enclosure. The remainder of the lamp output is lost energy. For any particular pumping scheme, a quantity η_G, the geometrical efficiency, can be defined as the ratio of the photon energy which enters the Laser rod to that produced by the pumping source, again within a specified wavelength range.

Capture Losses

Part of the energy which enters the Laser rod is used to pump ions from the ground state to the pumping band. The rest of the energy passes through the rod and is lost. The amount of energy lost in this manner depends on the dimensions of the rod, the density of active ions, and the degree to which the absorption spectrum of the active material matches the spectrum of the pumping source. The ratio of the energy transferred to active ions to the total energy entering the rod will be defined as the capture efficiency, η_C, of the system.

Relaxation Losses

When the active ions decay from the pumping band, some of their energy is expended in the desired transition from the metastable state to the terminal state, while the rest of the energy is dissipated in non-radiative transitions or radiated at frequencies other than the Laser output frequency. The ratio of the energy radiated in the desired transition to the total energy stored in the pumping band will be defined as the relaxation efficiency, η_Q. This quantity is equal to the quantum efficiency ε (the ratio of the number of photons radiated at the desired frequency to the total number of photons absorbed in the pumping process) multiplied by the ratio of the energy of an output photon to the average energy of the pumping photons.

Radiation Losses

Part of the energy available in the desired transition goes into the coherent radiation field, and the rest is lost through spontaneous emission. The magnitude of the radiation loss depends on how quickly threshold is reached and on the fluorescent lifetime of the metastable state. The corresponding radiation efficiency, η_R, is defined as the ratio of the energy which enters the coherent field to the total energy involved in the transition from the metastable state to the terminal state.

Optical Losses

Finally, some of the energy in the coherent field is channeled into useful output, while the rest is absorbed or scattered by the Laser rod and the end mirrors. The optical efficiency, η_O, can be defined as the ratio of useful output energy to the total energy which enters the coherent field.

Overall Efficiency

If the lamp efficiency, geometrical efficiency, capture efficiency, relaxation efficiency, radiation efficiency, and optical efficiency of a Laser system are defined as stated in the preceding paragraphs, then the overall system efficiency is simply the product of the six sub-efficiencies,

$$\eta = \eta_L \eta_G \eta_C \eta_Q \eta_R \eta_O \qquad (2\text{-}1)$$

III. Assumptions and Approximations

In order to keep the model from becoming enormously complicated, certain assumptions and approximations will be made concerning the system and its representation by the model. These assumptions and approximations fall into three general categories: (1) Assumptions about the geometry and physical characteristics of the system, (2) assumptions concerning the mathematical formulation of the efficiency equations, and (3) numerical approximations in the computer program.

Assumptions About the System

The Laser Rod. The active element in the Laser system will be assumed to be a right circular cylinder of some homogeneous material doped uniformly over its volume with an appropriate active ion. The ends of the rod will be assumed to be perfectly flat and perpendicular to the axis of the cylinder.

Illumination. It will be assumed that the Laser rod is illuminated uniformly over the cylindrical surface, that the effects of reflection and refraction at the surface can be neglected, and that the light flux entering the rod through any element of surface area dA is diffused evenly over a solid angle of 2π steradians within the rod. It is further assumed that the flux density in the rod is independent of distance along the axis and that the flux density can be calculated as if the rod were infinitely long, implying that either the rod length is much greater than the rod radius, or both ends have high reflectivity, or both.

It is believed that the above assumptions are reasonably valid for most solid state Lasers pumped by helical or annular lamps. The use of the model to represent other pumping schemes may require some modification of the first part of the computer program to adjust for the difference in flux distribution in the rod.

Assumptions Used in the Mathematical Development

The model described in this paper is intended to be applicable to a wide variety of Laser systems, and assumptions which are valid concerning some of these systems may be totally unacceptable when referred to others. For this reason, the computer operator is given a considerable degree of freedom to choose, by the way in which he uses the model, the assumptions which he wishes to make concerning the interrelation of various system parameters. Nonetheless, there are some assumptions inherent in the development from which there is no escape, and which must be understood if the results of the computer program are to be interpreted correctly. Some of the more important of these assumptions are listed below.

1. The population of the ground state is assumed to be equal at all times to the doping density; that is, depopulation of the ground state is negligible. This assumption is valid as long as the doping density is much greater than the inversion required for threshold.

2. The entire Laser rod is assumed to reach threshold as soon as the average population inversion in the rod is sufficient to sustain the dominant mode. The validity of this assumption may vary from one material or geometry to another, and should be remembered when results are interpreted.

3. It is assumed that the pumping pulse can be represented in time by the sum of two exponential terms. This assumption was made after studying the time traces of xenon lamp output pulses, and it was concluded that a time function containing two exponential terms is adequate for the description of the lamps studied. Its acceptability for other pumping sources has not been determined.

4. It is assumed that all coherent output is useful output, and no attempt is made to determine the distribution

of energy among the several modes which may be excited.

Approximations in the Computer Program

Graphical Integrations. At several points in the computer program it is necessary to evaluate definite integrals by graphical (numerical) means. All such integrations are carried out by using the trapezoidal rule,

$$\int_{x_1}^{x_2} f(x)dx = \Delta x\left[\frac{f(x_1)+f(x_2)}{2} + f(x_1+\Delta x) + f(x_1+2\Delta x) + \ldots + f(x_2-2\Delta x) + f(x_2-\Delta x)\right] \quad (3\text{-}1)$$

with one exception, a case where $f(x_1)$ is infinite. In this instance, the integral is evaluated as

$$\int_{x_1}^{x_2} f(x)dx = \Delta x\left[f(x_1+\Delta x/2) + f(x_1+3\Delta x/2) + \ldots + f(x_2-3\Delta x/2) + f(x_2-\Delta x/2)\right] \quad (3\text{-}2)$$

Although there are certainly more accurate graphical integration formulas than these, the errors arising from inaccurate integration are expected to be minor compared with errors in the experimental determination of the physical parameters of the system which must be entered as input data.

Representation of Spectra. Absorption and lamp output spectra are represented in the computer program by the values of capture cross-section and spectral power density at intervals of .01 microns (100 Å). If the actual spectra being represented have features which are not adequately described by such a representation, some adjustment of the input values from those experimentally determined may be necessary.

Threshold Time. The elapsed time between the beginning of the pumping pulse and the achievement of threshold enters into the calculation of radiation efficiency. In the computer program, the inversion population is examined for threshold condition at intervals of 20 microseconds, and the radiation efficiency is calculated using threshold time to the nearest 20 microseconds. For example, if threshold is actually achieved in 164 microseconds, the computer calculates the efficiency as if the time to threshold were 180 microseconds, since that is the first time checked for which the inversion population is sufficient to sustain the dominant mode.

Undoubtedly, a considerable improvement in accuracy could be effected by a careful revision of the computer program and the use of more exacting numerical analysis. It is felt, however, that the program as it stands will represent the mathematical model adequately, and that most of the errors encountered will arise from the inadequacy of the model itself. Perhaps the least certain assumptions of all are that a complex mechanism like the four-level Laser can be represented by anything as simple as the model developed in this paper, and that the physical parameters of the system can be measured with sufficient accuracy to yield meaningful results.

IV. The Efficiency Equations

In Chapter II, the overall efficiency of a four-level Laser system was defined as the product of six sub-efficiencies. The object of this chapter is to develop mathematical expressions for the sub-efficiencies in terms of measurable physical parameters of the system.

Independent Variables and Constants

For convenience, the system parameters will be grouped into two general classes. Those parameters which are more or less fixed by the choice of Laser material, pumping method, and operating environment will be called constants, and those which are more or less under the control of the design engineer will be called independent variables. From a mathematical standpoint, there is very little difference between these two classes of parameters; the main difference is reflected in the computer program itself by the fact that once the constants are entered, a number of problems can be run with different values of the independent variables without reentering the constants. This, of course, does not prevent the computer operator from changing constants whenever he feels it is necessary.

Independent Variables. The following system parameters have been selected as independent variables.

1. Laser rod length, ℓ, in centimeters.

2. Laser rod radius, R, in centimeters.

3. Active ion density, ρ, in ions per cubic centimeter.

4. End mirror reflectivities, R_1 and R_2, in watts per watt.

5. End mirror absorptivities, A_1 and A_2, in watts per watt.

6. Attenuation constant of the Laser rod material at the Laser output wavelength, α_0, in nepers per centimeter.

7. Input energy to the pumping source, E_1, in joules.

8. Pumping pulse decay time constant, τ_1, in seconds.

<u>Constants</u>. The following system parameters have been selected as constants.

1. Lamp efficiency, η_L, as defined in Chapter II.

2. Geometrical efficiency, η_G, as defined in Chapter II.

3. Lamp spectrum, $p(\lambda)$, in watts per centimeter.

4. Absorption cross-section, $\sigma(\lambda)$, in square centimeters.

5. Laser output wavelength, λ_0, in centimeters.

6. Quantum efficiency, ε, in photons per photon.

7. Lower and upper limits of absorption band, λ_1 and λ_2, in centimeters.

8. Energy difference between ground state and terminal state, ΔE, in joules.

9. Lifetime of the metastable state, τ, in seconds.

10. Fluorescent linewidth of the metastable-terminal transition, Δf, in cycles per second.

11. Ambient operating temperature, T, in degrees Kelvin.

12. If the lamp spectrum is to be represented by black body radiation, the temperature of the black body, T_L, in degrees Kelvin.

It may seem to the reader that listing the lamp efficiency and geometrical efficiency as constants is an oversimplification. Certainly the lamp efficiency will vary with the energy of the pulse, and the geometrical efficiency will change when the physical dimensions of the Laser rod are altered. However, while these two quantities are not really constant, neither is their relation to the rest of the system parameters capable of being expressed in mathematical terms. If the actual relationships between these quantities and the other parameters are known, it is possible to adjust for variations in lamp efficiency and geometrical efficiency by applying a correction factor to the value of lamp input energy entered into the program.

Capture Efficiency

Since the lamp efficiency and the geometrical efficiency have been defined as constants, the first step in the construction of the model is the development of an expression for capture efficiency. This will first be done for monochromatic light, calling the monochromatic capture efficiency W_r, and then extended to the general case involving a continuous spectrum of pumping energy.

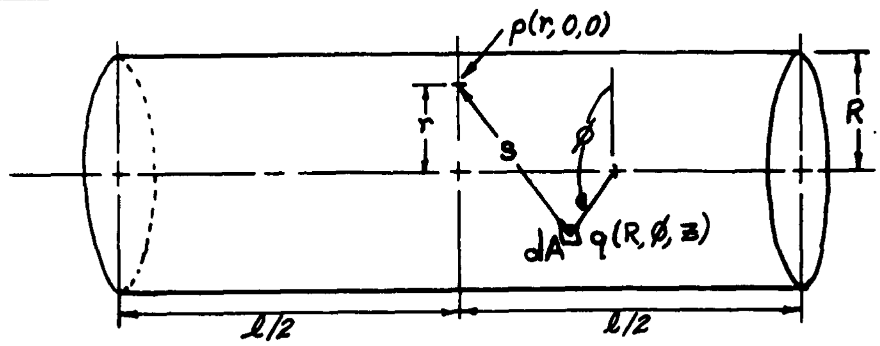

Fig. 2

Coordinate System for Development of the Model.

Consider a Laser rod having the geometry shown in Figure 2. Under the assumption (see Chapter III) that light entering the rod is uniformly spread over a solid angle of 2π steradians, the flux arriving at point $p(r,0,0)$ from an element of surface area dA located at point $q(R,\phi,z)$ is

$$d\psi = \frac{W}{2\pi s^2 A} e^{-\alpha s} \, dA \text{ watts/cm}^2 \tag{4-1}$$

where W is the total illuminating power, A is the total surface area, s is the distance from p to q, and α is an attenuation constant. In terms of the independent variables and constants defined earlier in this chapter,

$$A = 2\pi R\ell \quad cm^2 \tag{4-2}$$

$$s = (z^2+r^2+R^2-2rR\cos\phi)^{\frac{1}{2}} \quad cm \tag{4-3}$$

$$dA = R\,d\phi\,dz \quad cm^2 \tag{4-4}$$

$$\alpha = \rho\sigma(\lambda) \quad nepers/cm \tag{4-5}$$

Substituting (4-2), (4-3), and (4-4) in (4-1) and integrating over the surface gives the total flux at the point p as

$$\psi = \frac{W}{4\pi^2\ell}\int_{-\pi}^{+\pi}\int_{-\ell/2}^{+\ell/2}\frac{e^{-\alpha(z^2+r^2+R^2-2rR\cos\phi)^{\frac{1}{2}}}}{z^2+r^2+R^2-2rR\cos\phi}\,dz\,d\phi \quad watts/cm^2 \tag{4-6}$$

Assuming that $\ell \gg R$, or that both ends are nearly perfectly reflecting surfaces, and taking advantage of symmetry, (4-6) can be written

$$\psi = \frac{W}{\pi^2\ell}\int_0^{\pi}\int_0^{\infty}\frac{e^{-\alpha(z^2+r^2+R^2-2rR\cos\phi)^{\frac{1}{2}}}}{z^2+r^2+R^2-2rR\cos\phi}\,dz\,d\phi \tag{4-7}$$

The absorbed power density at any point in the rod is

$$dW_a = \alpha\psi \quad watts/cm^3 \tag{4-8}$$

and the total power absorbed is

$$W_a = \int_V \alpha\psi\,dV = 2\pi\ell\alpha\int_0^R \psi r\,dr \quad watts \tag{4-9}$$

The monochromatic capture efficiency, by definition, is W_a/W. Substituting (4-7) and (4-9) in this relation gives

$$W_r = \frac{2\alpha}{\pi} \int_0^R r \int_0^\pi \int_0^\infty \frac{e^{-\alpha(z^2+r^2+R^2-2rR\cos\phi)^{\frac{1}{2}}}}{z^2+r^2+R^2-2rR\cos\phi}\, dz\, d\phi\, dr \quad (4\text{-}10)$$

Equation (4-10) can be simplified considerably by change of variables. Let

$$R' = r/R \quad (4\text{-}11)$$

$$a = (r^2+R^2-2rR\cos\phi)^{\frac{1}{2}} \quad (4\text{-}12)$$

$$\gamma = \arctan(z/a) \quad (4\text{-}13)$$

$$a' = a/R = (1+R'^2-2R'\cos\phi)^{\frac{1}{2}} \quad (4\text{-}14)$$

When (4-11) through (4-14) are substituted in (4-10) and the results simplified, the expression for W_r becomes

$$W_r = \frac{2\alpha R}{\pi} \int_0^1 R' \int_0^\pi 1/a' \int_0^{\pi/2} e^{-(\alpha R a' \sec\gamma)}\, d\gamma\, d\phi\, dR' \quad (4\text{-}15)$$

Equation (4-15) completely specifies W_r, the monochromatic capture efficiency, in terms of one parameter, αR. Equation (4-5) shows, however, that α is in turn a function of the wavelength λ. Hence W_r is itself a function of λ.

If the spectral power density of the pumping source is $p(\lambda)$, then the total power in the spectrum between the limits λ_1 and λ_2 is

$$W = \int_{\lambda_1}^{\lambda_2} p(\lambda)\, d\lambda \quad \text{watts} \quad (4\text{-}16)$$

The power absorbed by the Laser rod is

$$W_a = \int_{\lambda_1}^{\lambda_2} W_r(\lambda)\ p(\lambda)\ d\lambda \quad \text{watts} \tag{4-17}$$

Further, if $p(\lambda)$ is normalized so that its average value over the range from λ_1 to λ_2 is unity, then (4-16) becomes

$$W = \lambda_2 - \lambda_1 \quad \text{watts} \tag{4-18}$$

and the capture efficiency is given by

$$\eta_C = W_a/W = \frac{1}{\lambda_2 - \lambda_1} \int_{\lambda_1}^{\lambda_2} W_r(\lambda)\ p(\lambda)\ d\lambda \tag{4-19}$$

Relaxation Efficiency

In Chapter II, it was established that the relaxation efficiency is equal to the quantum efficiency ε multiplied by the ratio of the energy of an output photon to the average energy of the pumping photons. Since the energy of a photon is inversely proportional to its wavelength, the relaxation efficiency is also equal to

$$\eta_Q = \varepsilon \frac{\lambda_p}{\lambda_o} \tag{4-20}$$

where λ_p is the wavelength of a photon whose energy is the average energy of the pumping photons.

For convenience, let the unit of energy in the following discussion be the energy associated with a photon of wavelength one centimeter ($1\ cm^{-1}$). Then the photon density associated with a spectral power density $p(\lambda)$ is

$$n(\lambda) = \lambda p(\lambda) \text{ photons/ cm·sec} \qquad (4\text{-}21)$$

The total number of photons absorbed per second is

$$N_a = \int_{\lambda_1}^{\lambda_2} W_r(\lambda)\, n(\lambda)\, d\lambda \text{ photons/sec} \qquad (4\text{-}22)$$

and the total power absorbed is

$$W_a = (\lambda_2 - \lambda_1)\eta_C \text{ cm}^{-1}\text{/sec} \qquad (4\text{-}23)$$

The average energy per photon is W_a/N_a, and the wavelength of the corresponding "average" photon is the reciprocal of the energy,

$$\lambda_p = N_a/W_a = \frac{1}{(\lambda_2-\lambda_1)\eta_C} \int_{\lambda_1}^{\lambda_2} \lambda p(\lambda) W_r(\lambda)\, d\lambda \text{ cm} \qquad (4\text{-}24)$$

Since the capture efficiency appears in the denominator of the expression for λ_p, it will also appear in the denominator of the expression for relaxation efficiency. Further, since the model makes no direct use of either η_C or η_Q individually, considerable time and effort can be saved by computing the product $\eta_C\eta_Q$ directly. Combining equations (4-20) and (4-24),

$$\eta_C\eta_Q = \frac{\varepsilon}{(\lambda_2-\lambda_1)\lambda_0} \int_{\lambda_1}^{\lambda_2} \lambda p(\lambda) Wr(\lambda)\, d\lambda \qquad (4\text{-}25)$$

<u>Radiation Efficiency</u>

The radiation efficiency was defined in Chapter II as the ratio of the energy which enters the coherent field to the total energy involved in the transition from the metastable state to the terminal state. Since all photons emitted by ions making this transition are of the same energy, the radiation efficiency is also equal to the ratio of the number of photons contributing to the coherent field to the total number emitted at the Laser output wavelength.

When a pulse of energy from the pumping source is applied to the four-level Laser, the population of the metastable state increases with time until, if the pumping energy is sufficiently high, a point is reached where the gain of the system for the coherent field exceeds unity. During the buildup of population, some ions are also decaying spontaneously to the terminal state, and the energy associated with the transition of these ions is lost. After the threshold has been reached, however, the coherent field builds up very rapidly, and nearly all of the ions which make the transition thereafter contribute their energy to the coherent field.

To determine the radiation efficiency, then, it is necessary to (1) obtain an expression for the population of the metastable state as a function of time, (2) determine the population required for threshold, (3) find the time for which the two are equal, and (4) calculate the amount of energy lost prior to threshold and thence obtain the radiation efficiency.

Metastable State Population. Assume (see Chapter III) that the pumping pulse can be represented by the sum of two exponential terms,

$$W(t) = C_1(e^{-t/\tau_1} - e^{-t/\tau_2}) \quad \text{watts} \tag{4-26}$$

The constant C_1 can be found by imposing the condition

$$\int_0^\infty W(t)\,dt = E_1\eta_L \quad \text{joules} \tag{4-27}$$

When this is done, the value of C_1 is obtained,

$$C_1 = \frac{E_1\eta_L}{(\tau_1-\tau_2)} \tag{4-28}$$

The equation for the population of the metastable state is

$$\frac{dn}{dt} = KW(t) - n/\tau \quad \text{ions/sec} \tag{4-29}$$

where

$$K = \frac{\eta_G\eta_C\eta_Q\lambda_0}{hc} \quad \text{ions/watt}\cdot\text{sec} \tag{4-30}$$

If (4-26), (4-28), and (4-30) are substituted in (4-29), the resulting equation can be solved for n, giving

$$n = \frac{E_1\eta_L\eta_G\eta_C\eta_Q\lambda_0}{hc(\tau_1-\tau_2)}\left\{\frac{\tau_1 e^{-t/\tau_1}}{(\tau_1-\tau)} + \frac{\tau_2 e^{-t/\tau_2}}{(\tau_2-\tau)} - \left[\frac{\tau_1}{(\tau_1-\tau)} + \frac{\tau_2}{(\tau_2-\tau)}\right] e^{-t/\tau}\right\} \quad \text{ions} \tag{4-31}$$

Since this equation represents the sum total of the population of the metastable state, the average density of ions in the metastable state is n/V, where V is the volume of the rod in cubic centimeters.

Threshold Condition. All of the preceding development applies equally well whether the Laser system is used as an oscillator, providing its own excitation, or as an amplifier, to build up an existing coherent field. The threshold conditions, however, are different for these two modes of operation, and since it is desired that the model represent both the Laser oscillator and the Laser amplifier, both threshold conditions will be developed.

When an electromagnetic wave of the proper frequency is passed through the Laser material, if the population of the metastable state exceeds the population of the terminal state, the wave will be amplified by the stimulated emission of radiation according to the equation

$$W(z) = W_0 e^{(\gamma - \alpha_0) z} \tag{4-32}$$

where z is the direction of propagation, α_0 is the attenuation constant of the material, and

$$\gamma = \sigma_{32} \Delta n \quad \text{nepers/cm} \tag{4-33}$$

In (4-33), Δn is the population difference between the metastable state and the terminal state in ions per cubic centimeter, and σ_{32} is the capture cross-section between the two states, given by (Ref 4:A-2)

$$\sigma_{32} = \frac{\lambda_0^2}{4\pi\tau\Delta f}\left[\frac{\ln(2)}{\pi}\right]^{\frac{1}{2}} \quad \text{cm}^2 \tag{4-34}$$

Applying equation (4-32) to the Laser oscillator of Figure 3, the following relations are obtained:

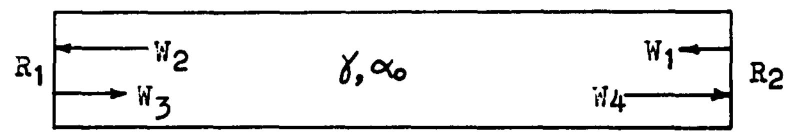

Fig. 3

Power Relations in the Laser Oscillator.

$$W_2 = W_1 e^{(\gamma-\alpha_0)\ell} \tag{4-35}$$

$$W_3 = R_1 W_2 = R_1 W_1 e^{(\gamma-\alpha_0)\ell} \tag{4-36}$$

$$W_4 = W_3 e^{(\gamma-\alpha_0)\ell} = R_1 W_1 e^{2(\gamma-\alpha_0)\ell} \tag{4-37}$$

If the gain of the system is unity, this also means that

$$W_1 = R_2 W_4 = R_1 R_2 W_1 e^{2(\gamma-\alpha_0)\ell} \tag{4-38}$$

The solution of (4-38) for γ gives

$$\gamma = \frac{2\alpha_0\ell - \ln(R_1 R_2)}{2\ell} \tag{4-39}$$

whence, by equation (4-33),

$$\Delta n = \frac{2\alpha_0\ell - \ln(R_1 R_2)}{2\sigma_{32}\ell} \tag{4-40}$$

The total ion density in the metastable state required for threshold is obtained by adding to Δn the thermal population of the terminal state, thus

$$n_t = \rho e^{-\Delta E/kT} + \frac{2\alpha_0 \ell - \ln(R_1 R_2)}{2\sigma_{32}\ell} \quad \text{ions/cm}^3 \qquad (4\text{-}41)$$

The threshold condition can be found from equations (4-31) and (4-41); that is, threshold is achieved when

$$\frac{n}{\pi R^2 \ell} = n_t \qquad (4\text{-}42)$$

Threshold for a Laser amplifier will be defined as the condition such that the net gain of the amplifier is unity. Neglecting reflections at the end surfaces, and applying equation (4-32),

$$\gamma = \alpha_0 \qquad (4\text{-}43)$$

$$n_t = \rho e^{-\Delta E/kT} + \alpha_0/\sigma_{32} \quad \text{ions/cm}^3 \qquad (4\text{-}44)$$

Again, equation (4-42) defines the required condition for threshold.

Threshold Time. The solution of equation (4-42) explicitly for time is a problem of considerable magnitude. Although this solution could be obtained by iteration to any desired accuracy, some compromise is necessary between obtaining an accurate value for threshold time and keeping the time required for calculation down to a reasonable level. As discussed in Chapter III, the actual procedure selected is to insert values of time at 20 microsecond intervals and check to see if threshold has been achieved. This process will be discussed more fully in Chapter V.

Computation of Radiation Efficiency. If the time to threshold is found to be T_1 seconds for a particular Laser system, the number of ions which will have decayed spontaneously prior to T_1 is given by

$$N_L = \int_0^{T_1} n/\tau \, dt \quad \text{ions} \tag{4-45}$$

Substituting the value of n from equation (4-31) and performing the integration gives

$$N_L = \frac{E_1\eta_L\eta_G\eta_C\eta_Q\lambda_o}{hc(\tau_1-\tau_2)}\left\{\frac{\tau_1^2}{\tau_1-\tau}(1-e^{-T_1/\tau_1}) + \frac{\tau_2^2}{\tau_2-\tau}(1-e^{-T_1/\tau_2}) - \left[\frac{\tau_1\tau}{\tau_1-\tau} + \frac{\tau_2\tau}{\tau_2-\tau}\right](1-e^{-T_1/\tau})\right\} \text{ions} \tag{4-46}$$

The total number of ions making the transition is

$$N_I = \frac{E_1\eta_I\eta_G\eta_C\eta_Q\lambda_o}{hc} \quad \text{ions} \tag{4-47}$$

and the radiation efficiency is given by

$$\eta_R = (N_I - N_L)/N_I \tag{4-48}$$

Substitution of (4-46) and (4-47) in (4-48) gives finally

$$\eta_R = 1 - \frac{1}{\tau_1-\tau_2}\left\{\frac{\tau_1^2}{\tau_1-\tau}(1-e^{-T_1/\tau_1}) + \frac{\tau_2^2}{\tau_2-\tau}(1-e^{-T_1/\tau_2}) - \left[\frac{\tau_1\tau}{\tau_1-\tau} + \frac{\tau_2\tau}{\tau_2-\tau}\right](1-e^{-T_1/\tau})\right\} \tag{4-49}$$

Equation (4-49) represents the radiation efficiency of a Laser oscillator; however, the radiation efficiency of a Laser amplifier will depend upon how much excitation is supplied at the input. If the exciting signal is very large, the efficiency will be fairly well represented by equation (4-49), and the gain will be very low. On the other hand, if the exciting signal is very small, the efficiency will be very low, and the gain will be higher. An investigation of the relation between gain and efficiency in the Laser amplifier is beyond the scope of this study. However, so that the model will be of some use to the designer of Laser amplifiers, two points on the gain-efficiency curve will be calculated, namely, the efficiency for unity gain and the gain for zero efficiency. These are labelled the maximum efficiency and maximum gain respectively, and it must be emphasized that the two cannot exist simultaneously.

The gain for very small signals, from equation (4-32), is

$$G_{max} = e^{(\sigma_{32}\Delta n_{max} - \alpha_0)\ell} \qquad (4\text{-}50)$$

where Δn_{max} is the maximum inversion achieved during the pumping period.

Optical Efficiency

With five of the six sub-efficiencies defined, either as constants or in terms of the system parameters, the only step remaining in the mathematical development of the efficiency equations is the derivation of an expression for the optical efficiency.

The optical losses in the system include both attenuation by the Laser rod itself and reflector losses at the ends of the rod (if the Laser is used as an oscillator). Referring again to Figure 3, page 25, and for convenience making the substitution

$$g = \gamma - \alpha_0 \tag{4-51}$$

the attenuation losses in the leftward travelling wave and the rightward travelling wave are respectively

$$W_{AL} = \int_0^{\ell} \alpha_0 W_1 e^{gx}\, dx \tag{4-52}$$

$$W_{AR} = \int_0^{\ell} \alpha_0 W_3 e^{gx}\, dx \tag{4-53}$$

When equations (4-36) and (4-51) are combined with (4-52) and (4-53) and the resulting equations integrated, the total loss due to attenuation is found to be

$$W_{LA} = W_{AL} + W_{AR} = \frac{\alpha_0 W_1}{g} (1 + R_1 e^{g\ell})(e^{g\ell} - 1) \tag{4-54}$$

The reflector losses at the left end and right end respectively are

$$W_{RL} = A_1 W_2 \tag{4-55}$$

$$W_{RR} = A_2 W_4 \tag{4-56}$$

Substitution of (4-35), (4-38), and (4-51) into the above gives the total reflector loss as

$$W_{LR} = W_{RL} + W_{RR} = W_1 (A_1 e^{g\ell} + A_2/R_2) \tag{4-57}$$

The total loss is

$$W_L = W_{LA} + W_{LR} \tag{4-58}$$

and the optical efficiency is

$$\eta_0 = W_0/(W_0 + W_L) \tag{4-59}$$

where W_0 is the output power, given by

$$W_0 = W_4(1-R_2-A_2) = \frac{W_1}{R_2}(1-R_2-A_2) \tag{4-60}$$

When (4-54) through (4-60) are combined and simplified, the resulting expression for optical efficiency becomes

$$\eta_0 = \frac{g(1-R_2-A_2)}{g(1-R_2) + \alpha_0 R_2 (1+R_1 e^{g\ell})(e^{g\ell}-1) + A_1 R_2 e^{g\ell}} \tag{4-61}$$

The value of g can be obtained from equations (4-39) and (4-51) as

$$g = \frac{-\ln(R_1 R_2)}{2\ell} \tag{4-62}$$

Since the maximum efficiency of the Laser amplifier is calculated with unity gain assumed, the power W is independent of distance along the rod, and the total loss (again neglecting reflections at the ends) is

$$W_L = \alpha_0 \ell W_0 \tag{4-63}$$

The optical efficiency of the Laser amplifier is then

$$\eta_0 = W_0/(W_0 + \alpha_0 \ell W_0) = 1/(1 + \alpha_0 \ell) \tag{4-64}$$

The so-called "walkoff" loss has been neglected in the derivation of the optical efficiency. Fox and Li (Ref 3:484) have shown that the walkoff loss for the dominant modes in a Fabry-Perot resonator is negligible compared with the attenuation and reflector losses. In the Laser amplifier, the severity of walkoff loss will be determined by the beam divergence of the exciting signal. If the input to the Laser amplifier is the output of a Laser oscillator, the walkoff loss will be very slight.

Summary

In this chapter, a mathematical model for the determination of Laser efficiency has been developed. The model consists of expressions for four of the six sub-efficiencies which were defined in Chapter II, in terms of the measurable physical parameters of the Laser system. These sub-efficiencies, when multiplied together and by the lamp efficiency and geometrical efficiency, which have been defined as constants, give the overall system efficiency of the four-level Laser system.

V. The Computer Program

The mathematical development of the Laser system model having been completed, the next step is the translation of the model into a program for the IBM 1620 computer.

Objectives

In the formulation of the computer program, it has been attempted to achieve certain objectives and conform to certain criteria in order to make the end product as useful as possible. Specifically, attainment of the following objectives was attempted:

1. The final program should be capable of being run on any IBM 1620 computer with card input/output, with no optional features required.

2. The program should represent as closely as possible the mathematical model developed in Chapter IV.

3. Execution of the program should be as convenient as possible. Wherever practicable, the program should be made self-explanatory by the use of typed out messages.

4. The program should produce results fast enough to be useful as a design aid; that is, during one reasonable period of operation, it should be possible to calculate the efficiency of several combinations of system parameters.

<u>Language</u>

The program has been written in a modification of the IBM FORTRAN language devised by 1st Lt. Pratt of the Air Force Institute of Technology Mathematics Department. There are a few differences between this language and standard FORTRAN, the more important from the programmer's standpoint being the following:

1. Format specification is not required on input statements, unless it is desired to prevent the reading of some of the data on a card.

2. The automatic typewriter carriage return at the beginning of each PRINT statement has been deleted, allowing a full 87 character line to be typed by two successive PRINT statements.

3. Trace instructions are automatically compiled, and the resulting object program is at least as short as the same program compiled in standard FORTRAN without trace instructions.

4. The subroutines have been shortened so that the object program begins at storage location 08000.

These differences have been noted here to indicate that some changes in the source program statements may be required if it is desired to compile the program developed in this chapter in standard FORTRAN.

To meet the first objective set forth above, the program must be capable of being run on a computer having a 20,000 digit memory. To meet this objective, it was necessary to split the program into three smaller programs which are run consecutively. The divisions have been arranged so that part 1 of the program is completely independent of the system parameters, and need only be run once. The output of part 2 is dependent only on certain of the system parameters which have been defined as constants, and this part need only be rerun when the constants involved are changed. Part 3 is that portion of the program which accepts the values of the independent variables and calculates the final results.

The intermediate data represented by the outputs of part 1 and part 2 of the program are stored on cards and read into the computer at the beginning of execution of the following part. Thus if it is desired to investigate the effect of changing certain variables in a system, and part 2 of the program has, at some previous time, been run using constants appropriate to the Laser material involved, it is only necessary to read the appropriate card deck into part 3 and enter the desired values of the variables.

PAGE 35 BLANK

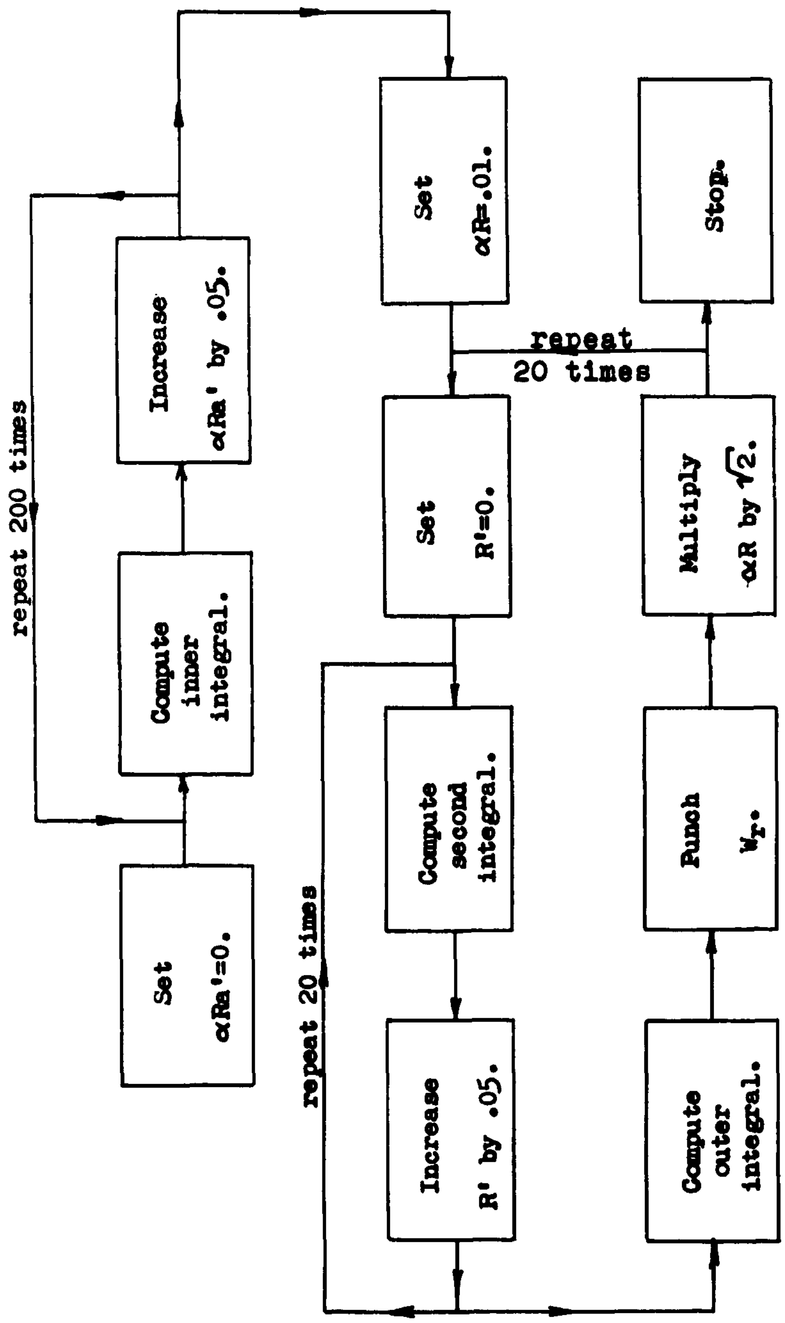

Fig. 4

Block Diagram of Part 1 of Computer Program.

Part 1

Part 1 of the computer program consists of the necessary instructions to effect the solution of equation (4-15),

$$W_r = \frac{2\alpha R}{\pi} \int_0^1 R' \int_0^{\pi} 1/a' \int_0^{\pi/2} e^{-(\alpha Ra' \sec\gamma)} d\gamma \; d\phi \; dR' \qquad (4\text{-}15)$$

A block diagram of the program is shown in Figure 4. The program first constructs a table of the values of the inner integral for values of the parameter $\alpha Ra'$. Integration is done by the trapezoidal rule, equation (3-1), and the interval $\Delta\gamma$ used is $\pi/100$. The integral is computed for values of $\alpha Ra'$ from zero to ten at intervals of 0.05, and the resulting values are stored as a subscripted variable. The next integration is carried out with αR and R' as parameters. Integration is accomplished using equation (3-2) (rectangular elements of area), and $\Delta\phi$ is $\pi/100$. R' is allowed to vary from zero to one in steps of 0.05, and αR takes on values from 0.01 through 10.24, each of which (except the first) is $\sqrt{2}$ times the preceding value. The last integration uses the trapezoidal rule, $\Delta R'$ being 0.05. The output consists of a set of values of W_r, one for each value of αR, punched one to a card (21 cards). The actual source statements used in the compilation of this and the other two parts of the program are listed in Appendix A.

Part 1 of the program requires 13 hours for execution, and since it has already been run, there is no need to run it again unless it is desired to alter the logic in some way. The required cards for input to part 2 may be punched manually, and if this is done, the cards should be punched in columns 2 through 14 as follows:

```
.19397179E-01
.27209617E-01
.38035933E-01
.52906180E-01
.73180046E-01
.10051629E+00
.13685365E+00
.18418166E+00
.24438795E+00
.31849994E+00
.40594415E+00
.50361047E+00
.60531516E+00
.70225637E+00
.78571970E+00
.84970604E+00
.89381930E+00
.92186415E+00
.93919806E+00
.94753762E+00
.94845226E+00
```

The monochromatic capture efficiency W_r ranges from about 2 percent for $\alpha R = 0.01$ to about 95 percent for $\alpha R = 10.24$. It is believed that this range suffices to represent most commonly used four-level Laser systems.

If it is desired to run this part of the program in the future, all that is required on the part of the operator is to load the program and press PUNCH START and START.

PAGE 39 BLANK

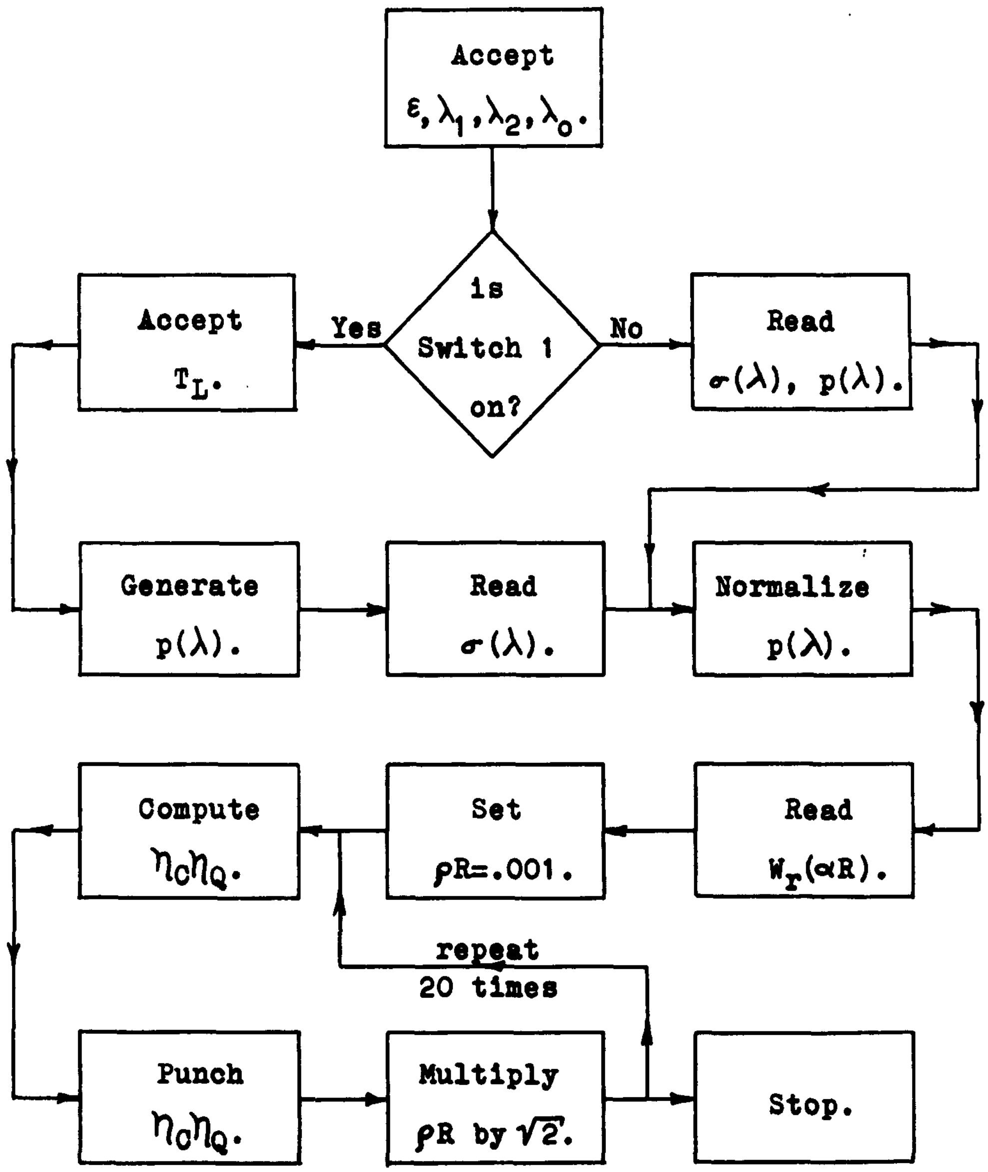

Fig. 5

Block Diagram of Part 2 of Computer Program.

During execution of the program, several error messages will be typed out, such as ERROR E4 and ERROR F5. These messages arise from computations of the form

$$y = e^{-(1/0)} \tag{5-1}$$

and should be ignored, as the correct value of y, namely, zero, results from this computation.

Part 2

Part 2 of the computer program accepts the output cards from part 1 and some of the system parameters which were defined as constants in Chapter IV, and computes the solution of equation (4-25),

$$\eta_c \eta_Q = \frac{\varepsilon}{(\lambda_2 - \lambda_1)\lambda_0} \int_{\lambda_1}^{\lambda_2} \lambda\, p(\lambda) W_r(\lambda)\, d\lambda \tag{4-25}$$

A block diagram of the program is shown in Figure 5. After accepting via the typewriter the values of ε, λ_1, λ_2, and λ_0, the program determines, from the setting of a console switch, whether the operator wishes to enter the lamp spectrum explicitly or allow it to be represented by black body radiation. In the former case, the program reads the values of σ and p for the range from λ_1 to λ_2 from cards which have been manually punched with the appropriate values. It then normalizes $p(\lambda)$ so that its average value is unity. In the latter case, only $\sigma(\lambda)$ is read from cards, and the computer generates the black body radiation function

$$p(\lambda) = \frac{K\lambda^{-5}}{e^{(hc/kT\lambda)}-1} \qquad (5\text{-}2)$$

at a temperature selected by the operator, where K is adjusted so that the average value of $p(\lambda)$ is unity.

Next, the program reads the output cards from part 1 and interpolates the table of W_r to obtain the correct value for each wavelength, with ρR as a parameter. ρR is allowed to vary from 0.001×10^{20} to 1.024×10^{20} cm^{-2}, with each value (except the first) being $\sqrt{2}$ times the preceding value. The range of σ which the program will handle without distortion varies with ρR. Values which result in αR less than 0.01 are assigned $W_r = 0$, and those which result in values of αR greater than 10.24 are assigned the highest value in the table, $W_r = 0.94845226$. The particular range selected was designed to fit the trivalent neodymium ion, which has a peak absorption cross-section of about 8×10^{-20} cm^2 (Ref 1:196).

The integration is performed by the trapezoidal rule with $\Delta\lambda = .01$ micron. The output consists of a value of $\eta_C \eta_Q$ for each value of ρR, punched one to a card (21 cards).

<u>Operation</u>. The operation of part 2 of the program is as follows: After the program is loaded and the START button pressed, the following message is typed:

SWITCH 1 ON TO REPRESENT LAMP SPECTRUM BY BLACK BODY RADIATION, PUSH START.

and the computer halts. After this instruction has been complied with, the following message is typed:

ENTER QUANTUM EFFICIENCY.

The quantum efficiency of the Laser material is entered as a decimal fraction. The following message is then typed:

ENTER LIMITS OF ABSORPTION SPECTRUM IN MICRONS.

The limits λ_1 and λ_2 are entered; the order of their entry is immaterial. The following constraints are placed on the values of these limits:

1. $\lambda_2 - \lambda_1$ must be an integral multiple of .01 microns.

2. $\lambda_2 - \lambda_1$ must not exceed 1 micron.

After these values have been entered, the following message is typed:

ENTER LASER OUTPUT WAVELENGTH IN MICRONS.

λ_0 is then entered.

At this point, if console switch 1 is on, the following message is typed:

ENTER BLACK BODY TEMPERATURE IN DEGREES K.

T_L is entered, and the black body radiation function is generated. The program will then read from cards the values of $\sigma(\lambda)$. The input cards must contain ONLY the values of σ in units of 10^{-20} cm^2, punched one to a card, and arranged in order of increasing wavelength from λ_1 to λ_2 inclusive. Each card read will be associated with a wavelength 0.01 microns longer than the previous card, and the program will read exactly $\frac{\lambda_2-\lambda_1}{.01}$ + 1 cards.

If console switch 1 is off, the program will read from cards the values of $\sigma(\lambda)$ and $p(\lambda)$. The cards must be punched with two numbers each, the first number being $\sigma(\lambda)$ in 10^{-20} cm^2 and the second being the spectral power density p in any units desired (the computer will normalize the spectrum). The wavelength value associated with each card and the order of the cards is as described in the preceding paragraph.

At this point, the output cards from part 1 will be read, and the remainder of the program will proceed without further intervention by the operator. The total running time for this part of the program (using an absorption band 0.6 microns wide) is about 25 minutes.

Part 3

Part 3 of the program accepts the output data from part 2 and the remaining constants and independent variables, and computes either the efficiency of a Laser oscillator or the maximum efficiency and maximum gain of a Laser amplifier,

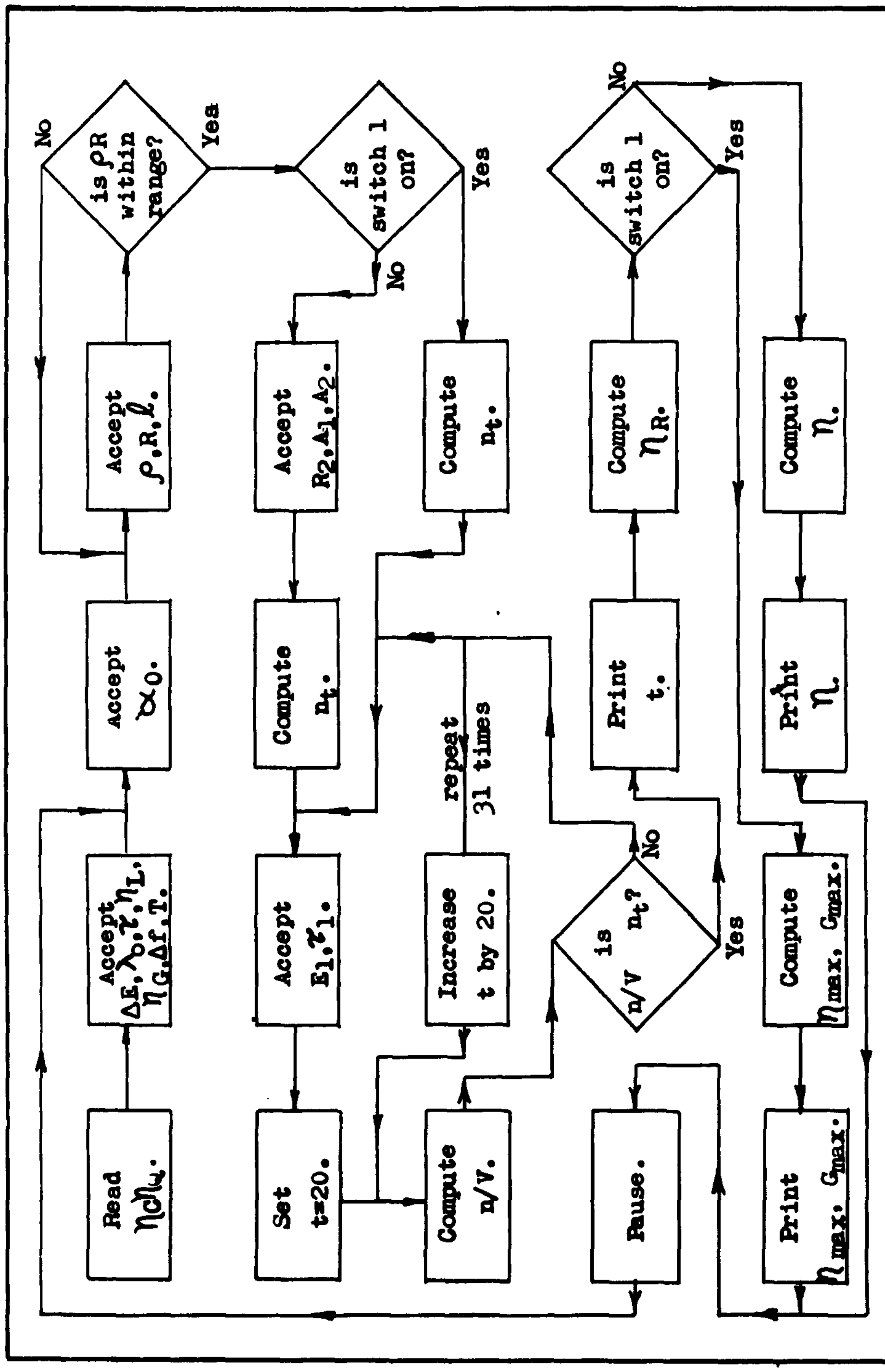

Fig. 6

Block Diagram of Part 3 of Computer Program.

depending on the setting of a console switch. A block diagram of part 3 is shown in Figure 6.

At the beginning of the program, the output cards from part 2 are read, the constants ΔE, λ_o, τ, η_L, η_G, Δf, and T are accepted, and some intermediate constants are computed to save time later in the program. The variables α_o and ρ are accepted, and the thermal population of the terminal state,

$$p_t = \rho e^{-\Delta E/kT} \tag{5-3}$$

is computed. The variables R and ℓ are accepted, and a check is made to see if $.001 \leq \rho R x 10^{-20} \leq 1.024$. If not, new values of ρ, R, and ℓ are requested.

At this point, if console switch 1 is on, the population required for threshold is computed from equations (4-44) and (5-3) as

$$n_t = p_t + \alpha_o/\sigma_{32} \tag{5-4}$$

If the switch is off, R_2, A_1, and A_2 are accepted. R_1 is assumed to be $1 - A_1$ (no transmission through one end), and the threshold population is calculated using equations (4-41) and (5-3),

$$n_t = p_t + \frac{2\alpha_o \ell - \ln(R_1 R_2)}{2\sigma_{32}\ell} \tag{5-5}$$

The values of E_1 and τ_1 are then accepted. The value of τ_2 is calculated as $\tau_1/10.35$. This ratio was arrived at on the basis of oscilloscope traces of the output of a General Electric FT-506 helical xenon flash lamp, and is the ratio of time constants which produces a power maximum at time $t = \frac{1}{4}\tau_1$. Some of the oscilloscope photographs used as the basis for the selection of the time constant ratio are shown in Figure 7. The traces shown are all of 500 joule pulses, the input voltages and capacitances being, from top to bottom,

(1) 1225 v, 672 μf.
(2) 1000 v, 1008 μf.
(3) 867 v, 1344 μf.
(4) 735 v, 1680 μf.

The time scale is 1 cm=.5 ms. If for any reason it is desired to change the ratio, it is only necessary to reflect the change in statement number 120 in the source program (see Appendix A).

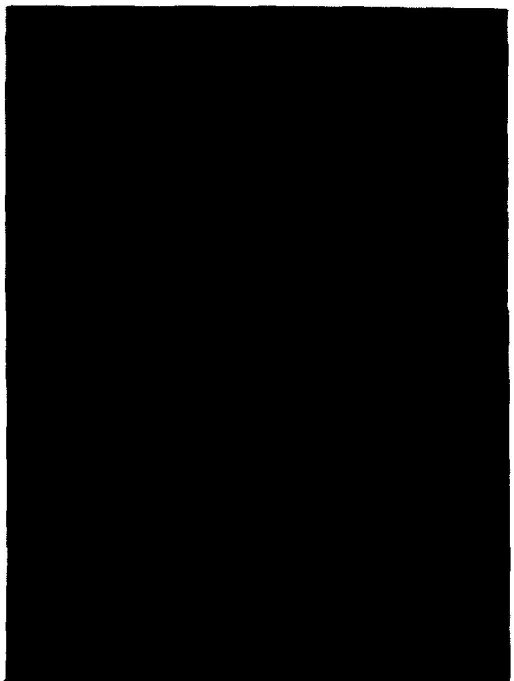

Fig. 7

Typical Output Waveforms of the G.E. FT-506 Lamp.

It is interesting to note that for the lamp checked, τ_1 in microseconds is approximately equal to the capacitance in microfarads.

Having obtained the values of all of the system parameters, the program interpolates the table of $\eta_C \eta_Q$ to obtain the correct value for the radius and ion density entered, and proceeds to calculate the metastable state population at intervals of 20 microseconds, comparing each result with the value of n_t. If threshold is not reached in 640 microseconds, a message is typed indicating that more energy is required. The time for which the value of the metastable state population (n/V) first exceeds or equals n_t is considered to be T_1, the threshold time, and either the efficiency, equations (4-49), (4-61), and (2-1), or the maximum efficiency and maximum gain, equations (4-49), (4-64), (2-1), and (4-50), are computed, depending on the setting of console switch 1. In the computation of maximum gain, the metastable population is computed at 20 microsecond intervals until a value is found which is less than the previous value. The higher value is then used to compute the maximum gain. If the population is still increasing at $t = 640$ microseconds, the value of n corresponding to $t = 640$ microseconds is used in the computation.

After typing the results, the computer will halt. Pressing the START button returns the program to the point where α_0 was entered, and the cycle repeats.

Operation. The operation of part 3 of the program is as follows: After the program has been loaded and the START button pressed, the output cards from part 2 are read and the following message is typed:

ENTER ENERGY DIFFERENCE BETWEEN GROUND STATE AND TERMINAL STATE IN RECIPROCAL CM.

The value of ΔE is entered, and the following message is typed:

ENTER LASER OUTPUT WAVELENGTH IN MICRONS.

λ_0 is entered, and the following message is typed:

ENTER LIFETIME OF METASTABLE STATE IN MICROSECONDS.

τ is entered, and the following message is typed:

ENTER LAMP EFFICIENCY, GEOMETRICAL EFFICIENCY.

η_L and η_G are entered. It should be remembered that in the determination of η_L, only the portion of the spectrum between λ_1 and λ_2 is considered as useful output. After these values have been entered, the following message is typed:

ENTER FLUORESCENT LINE WIDTH IN CYCLES PER SECOND.

Δf is entered, and the following message is typed:

ENTER OPERATING TEMPERATURE IN DEGREES K.

T is entered, and the following message is typed:

ENTER ATTENUATION CONSTANT OF HOST MATERIAL.

α_0, in nepers/cm, is entered, and the following message is printed:

ENTER DOPING DENSITY IN IONS PER CC.

ρ is entered, and the following message is typed:

ENTER ROD RADIUS AND LENGTH IN CM.

R and ℓ are entered, and a check is made to see if ρR is within the acceptable range. If not, the following message is typed:

RHO R OUT OF RANGE
ENTER DOPING DENSITY IN IONS PER CC.

As soon as an acceptable combination of ρ and R has been obtained, the following message is typed:

SWITCH 1 ON FOR AMPLIFIER, OFF FOR OSCILLATOR,
PUSH START.

If console switch 1 is set to the ON position, the remainder of the program will calculate the maximum efficiency and maximum gain of a Laser amplifier. If the switch is set to OFF, the program will calculate the efficiency of a Laser oscillator.

After the switch has been set and the START button depressed, if switch 1 is off, the following message is typed:

ENTER REFLECTIVITY OF OUTPUT END.

R_2 is entered, and the following message is typed:

ENTER ABSORPTIVITY OF CLOSED END, OUTPUT END.

A_1 and A_2 are entered. If switch 1 is on, the reflectivities and absorptivities are not required by the program (assumed zero), and the preceding two messages are omitted. In any event, the following message is then typed:

ENTER LAMP ENERGY IN JOULES AND DECAY TIME CONSTANT IN MICROSECONDS.

E_1 and τ_1 are entered, the computer proceeds to compute the metastable population, and the results are compared with the threshold condition. If threshold is not reached, the following message is typed:

THRESHOLD NOT REACHED, INCREASE LAMP ENERGY.
ENTER LAMP ENERGY IN JOULES AND DECAY TIME CONSTANT IN MICROSECONDS.

When a value of lamp energy has been obtained which produces the threshold condition, the following message is typed:

TIME TO THRESHOLD = XXX MICROSECONDS

The efficiency (switch 1 off) or maximum efficiency and gain (switch 1 on) are computed, and the results are printed in the appropriate one of the following two formats:

EFFICIENCY = X.XXXXXXXX PERCENT.

MAX. EFFICIENCY = X.XXXXXXXX PERCENT, MAX. GAIN = XXX.XXXXXXX

After printing the results, the computer halts. A new set of independent variables may be entered by pressing the START button. The running time for part 3 is about eight minutes for the first set of parameters, and two to three minutes for each subsequent set of independent variables, including time required to manually enter the required data.

Summary

In this chapter, a computer program has been described which calculates the efficiency of a Laser oscillator or the maximum efficiency and maximum gain of a Laser amplifier from the values of the system parameters. The program can be run on any IBM 1620 computer which has card input/output, and follows closely the mathematical model developed in Chapter IV. It is convenient to use, and has been made almost completely self-explanatory by the use of typed out messages. After about 30 minutes of precomputation, the program will process a set of independent variables and print the results in two to three minutes.

VI. A Sample Problem

This chapter is an illustration of how the computer model developed in the preceding chapters might be applied to a design problem. It is not intended to be an actual analysis of any specific Laser system, existing or proposed, although an effort has been made to make the values of the constants used in the sample problem conform as closely as possible to the known characteristics of the materials involved. In some cases, these constants were actually measured by the author, in other cases they were extracted from the literature, and in still others, they represent pure guesswork.

Nonetheless, it is believed that all of the system parameters used in the model are physically measurable, and it would be interesting to actually measure these quantities for a real Laser system and compare the measured efficiency under various conditions with the efficiency predicted by the model. Unfortunately, such a comparison is beyond the scope of this study (see Appendix B).

The Problem

It is desired to design a Laser oscillator consisting of a rod of glass doped with trivalent neodymium and pumped by an annular xenon flash lamp. The inner surface of the lamp is to be in contact with the rod, and under these conditions the maximum energy per pulse allowed will be given by

$$E_{1_{max}} = 100R\ell \text{ joules} \tag{6-1}$$

The lamp efficiency in the range .3 to .9 microns is 25%, and the geometrical efficiency of the pumping scheme is 72%. It is desired to maximize the energy output of the Laser, subject to the following restrictions:

1. The length of the rod will be 25 cm.
2. The minimum radius of the rod is 0.3 cm.
3. The maximum radius of the rod is 1.2 cm.
4. The doping density is 7×10^{19} ions/cm^3.
5. The reflectivity of the output end, to achieve the desired mode selection, must be at least 50%.

Assume that the ends of the rod are to be coated with multiple layers of dielectric, and that the characteristics of these surfaces are given by

$$A_n = 0.002R_n, \quad n = 1, 2 \tag{6-2}$$

and that the lamp decay time constant τ_1 is given by

$$\tau_1 = 4E_1 \times 10^{-7} \text{ seconds} \tag{6-3}$$

Further assume that the lamp spectrum can be fairly well represented by black body radiation at 7000°K.

A sample of the Laser material is produced, and measurements made which indicate the following values for the physical properties of the material:

$$\lambda_0 = 1.062 \text{ microns}$$

$$\varepsilon = 0.34$$

$$\Delta E = 1930 \text{ cm}^{-1}$$

$$\tau = 0.48 \text{ ms}$$

$$\Delta f = 1.19 \times 10^{13} \text{ cps}$$

$$\alpha_0 = 0.015 \text{ nepers/cm}$$

$\sigma(\lambda)$ as given in Table I

Table I
Absorption Cross-sections of Material in 10^{-20} cm^2.

λ	σ	λ	σ	λ	σ	λ	σ
.30	0.	.46	0.027	.61	0.300	.76	1.460
.31	0.	.47	0.030	.62	0.	.77	0.352
.32	0.	.48	0.033	.63	0.	.78	0.278
.33	0.	.49	0.	.64	0.	.79	1.050
.34	0.	.50	0.200	.65	0.	.80	3.380
.35	0.	.51	0.740	.66	0.	.81	5.750
.36	0.	.52	0.860	.67	0.053	.82	1.300
.37	0.	.53	1.810	.68	0.214	.83	0.374
.38	0.	.54	0.270	.69	0.181	.84	0.032
.39	0.	.55	0.	.70	0.	.85	0.032
.40	0.	.56	0.064	.71	0.	.86	0.342
.41	0.	.57	1.300	.72	0.032	.87	0.779
.42	0.	.58	5.820	.73	0.370	.88	1.410
.43	0.117	.59	8.000	.74	3.230	.89	0.565
.44	0.	.60	1.880	.75	3.760	.90	0.235
.45	0.075						

The values of absorption cross-section in Table I are those actually measured by the author for a neodymium doped lead glass sample furnished by the Isomet Corporation, using a Cary Model 14 spectrophotometer. The values of τ and Δf were obtained from a paper describing experiments with neodymium doped barium crown glass at the American Optical Company (Ref 2:364, 366). The value of α_0 corresponds to a fifty percent loss in about 46 cm. The quantum efficiency is admittedly a guess on the part of the author.

If it is now assumed that the Laser rod will be operated at room temperature (300°K), enough information is available to compute efficiency as a function of radius and output end reflectivity. Multiplication of the resulting efficiencies by the appropriate lamp energies will give the output energies.

Solution

The parameters of the system described above were entered into the computer model and the efficiency and output energy calculated for ten values of R between .3 cm and 1.2 cm, and for five values of R_2 between 0.5 and 0.9. The resulting efficiencies and output energies are listed in Tables II and III on the following page. The total time required to compute the fifty efficiencies, including the time required to warm up the computer and load the programs, was slightly less than two and one-half hours.

Table II
Percent Efficiency vs. Radius and Reflectivity.

R(cm)	R_2=.5	.6	.7	.8	.9
.3	.286	.241	.193	.137	.073
.4	.335	.282	.224	.159	.084
.5	.373	.315	.251	.178	.095
.6	.405	.341	.273	.193	.103
.7	.428	.363	.290	.206	.110
.8	.447	.382	.306	.218	.116
.9	.462	.396	.317	.226	.121
1.0	.465	.407	.328	.234	.125
1.1	.446	.408	.336	.242	.129
1.2	*	.400	.339	.246	.133

*Threshold not reached.

Table III
Output Energy in Joules vs. Radius and Reflectivity.

R(cm)	R_2=.5	.6	.7	.8	.9
.3	2.14	1.81	1.45	1.03	0.55
.4	3.35	2.82	2.24	1.59	0.84
.5	4.66	3.94	3.14	2.23	1.19
.6	6.08	5.12	4.09	2.90	1.54
.7	7.49	6.35	5.08	3.60	1.93
.8	9.94	7.64	6.12	4.36	2.32
.9	10.86	9.31	7.45	5.31	2.84
1.0	11.63	10.18	8.20	5.85	3.13
1.1	12.26	11.22	9.25	6.65	3.55
1.2	0.00	12.00	10.17	7.38	3.99

An examination of the tables shows that the maximum output energy is obtained when the radius is 1.1 cm and the output end reflectivity is 0.5. A Laser designed using these values of R and R_2 should produce 12.26 joules of output with an overall system efficiency of 0.446 percent. The maximum efficiency, on the other hand, is given by making the radius 1.0 cm and the output end reflectivity 0.5. A Laser built using these values should produce 11.63 joules of output with an efficiency of 0.465 percent.

One feature of the computer model which is not evident from Tables II and III is that the computer prints the value of T_1, the time to threshold, for each set of variables entered. This gives the operator a qualitative idea of how close to threshold the Laser system would operate under given conditions. In the sample problem above, the designer might wish to maintain a safety factor to insure that the Laser constructed would actually reach threshold even though some errors might be present in the measurement of the parameters or in the assumptions made.

One way to do this is to restrict the acceptable combinations of R and R_2 to those which reach threshold in less than a certain specified time period. If, as an example, no solutions are considered for which T_1 is greater than 200 microseconds, the combinations of R and R_2 located below and to the left of the dashed lines in Tables II and III must be eliminated from consideration. If this restriction is adopted, the maximum efficiency (0.462%) and maximum output energy (10.86 j) both occur when R=0.9 and R_2=0.5.

In the sample problem above, the efficiency and output were calculated for all possible combinations of the two variables being optimized. This was done merely to illustrate the variation in efficiency and output over the entire range of possible combinations of R and R_2. In an actual design application, it would probably be quicker and more satisfactory to use a "hill-climbing" method of optimization, particularly if several parameters were to be allowed to vary. In the problem above, the optimum values of R and R_2 could have been obtained in fifteen tries rather than fifty, even if the worst possible starting point had been selected and all possible mistakes in judgement made. Care must be exercised in the use of this method, however, to avoid ending on top of the wrong "hill". If the 200 microsecond maximum is imposed on T_1, it is possible to obtain, by the hill-climbing method, an optimum solution of R=1.2 and R_2=.8, for an output of only 7.38 joules at an efficiency of 0.246 percent. Even without the limit on T_1, an optimum solution of R=1.2, R_2=.6 might be reached, giving an output of 12 joules at 0.4 percent efficiency. The more closely spaced are the test values of the variables, the less is the chance of arriving at a false maximum by the hill-climbing method.

Summary

In this chapter, the use of the computer model was illustrated by the solution of a hypothetical design problem in which some of the independent variables were assigned fixed values, others were constrained to lie within certain limits, and still others were made functions of other variables. The flexibility of the model was demonstrated by optimizing the variables with respect to output energy rather than efficiency, and by introducing the possibility of maintaining a safety factor to insure the achievement of threshold. Fifty trial solutions were obtained, and the total computer operating time for part 2 and part 3 of the program was less than two and one-half hours. The use of "hill-climbing" techniques in the optimization process was suggested as a means for making the solution even faster, although it was pointed out that this method may lead to false solutions if care is not exercised.

The computer printout obtained in the solution of the sample problem is included in this paper as Appendix C.

VII. Results and Conclusions

The main result of the study described in this paper is, of course, the mathematical model and associated computer program for the determination of four-level Laser efficiency. Unfortunately, time limitations have prevented the application of the model to any actual problems existing today in the Laser design field. Nonetheless, some conclusions may be drawn from the model and from the experience gained in its development and testing regarding the probable effect of changing each of the independent variables in a Laser system on the system efficiency. Some of these effects have already been physically observed, and this fact lends support to the validity of the model. In general, it can be said that, within the limitations imposed by the assumptions and approximations used in the development of the model, the system efficiency will:

1. Increase as the Laser rod length is increased.
2. Increase as the Laser rod radius is increased.
3. Increase as the doping density is increased.
4. Decrease as the end reflectivities are increased.
5. Increase as the pumping energy is increased.
6. Decrease as the pumping pulse decay time constant is increased.

No attempt has been made to make the model applicable to continuous operation of the four-level Laser, or to the so-called pulsed reflector and hair-trigger modes of operation. It is believed, however, that the basic mathematical development is applicable to all of these conditions, and that only minor changes in the computer program would be necessary in order to be able to represent them by the model.

Some thought was given to the possibility of making the program self-optimizing; that is, writing the program in such a way that it would perform automatically the hill-climbing optimization referred to earlier with respect to any or all of the independent variables, within limits set for each variable, and print as output not only the maximum obtainable efficiency, but also the optimum values for the variables. It was felt, however, that this approach could not be justified, for the following reasons:

1. The time required for the 1620 computer to accomplish an adequate optimization by the hill-climbing method, particularly if many variables were allowed to change and small increments of each variable were used, would probably be quite long.

2. It would be very difficult to make the program general enough to allow for interrelations among the

variables, such as those postulated between rod dimensions and lamp parameters in the sample problem of Chapter VI.

3. The previously mentioned possibility of arriving at a false optimum point through the use of hill-climbing techniques would require the results of a self-optimizing program to be interpreted with extreme skepticism.

4. The present program (part 3) uses most of the memory of the 1620, and it is doubtful that self-optimization could be included in a program which would fit in a 20,000 digit memory.

In conclusion, it should be stressed again that the computer model described in this paper has not been checked against any experimental results, although it is known that it does predict the correct order of magnitude of efficiency and threshold for neodymium doped glass Lasers. Before the program is used in actual design problems, it is strongly recommended that at least some experimental verification of its adequacy be obtained. This project, along with others which the author feels are necessary in the Laser field, is included in Appendix B, a list of suggested topics for future research.

Bibliography

1. Boyle, W. S., and D. F. Nelson. "Continuously Pumped Optical Masers," in Aerospace Electronics 1962 National Conference Proceedings. Dayton, Ohio: Institute of Radio Engineers, 1962, pp. 194-200.

2. Dixon, E., and E. Snitzer. "Optical Maser Program at American Optical Company," in Aerospace Electronics 1962 National Conference Proceedings. Dayton, Ohio: Institute of Radio Engineers, 1962, pp. 364-368.

3. Fox, A. G., and T. Li. "Resonant Modes in a Maser Interferometer." Bell System Technical Journal, 40:453-488 (March 1961).

4. Hughes Research Laboratories. Research on Coherent Generation of Optical Radiation. Interim Scientific Report No. 4, Contract No. AF 33(616)-8233, 1 February 1962 through 30 April 1962. Malibu, California: Hughes Research Laboratories, no date.

Appendix A

FORTRAN Program

PART 1

```
C     PART 1, 4-LEVEL LASER EFFICIENCY AND GAIN PRØGRAM (IBM 1620).
C     THIS PRØGRAM CALCULATES THE CAPTURE EFFICIENCY VS. RADIUS TIMES
C     ATTENUATIØN CØNSTANT.
C     RUNNING TIME APPRØXIMATELY  780 MINUTES
      DIMENSIØN F(201),FRP(21)
    1 FØRMAT(E14.8)
      SR=SQRT(2.)
      BETA=0.
      DØ 101 I=1,201
      F(I)=.015707963*EXP(-BETA)
      GAMMA=.031415927
      DØ 102 J=1,49
      F(I)=F(I)+.031415927*EXP(-BETA/CØS(GAMMA))
  102 GAMMA=GAMMA+.031415927
  101 BETA=BETA+.05
      ALPHR=.01
      DØ 110 J=1,21
      RP=C.
      DØ 109 K=1,21
      PHI=.015707963
      FRP(K)=0.
      N=2
      A=.05
      DØ 106 L=1,100
      APR=SQRT(1.+RP**2-2.*RP*CØS(PHI))
      BETA=ALPHR*APR
      DØ 124 I=N,201
      IF(BETA-A)125,126,124
  125 FØFA=F(I)-(F(I)-F(I-1))*(A-BETA)/.05
      GØ TØ 127
  126 FØFA=F(I)
      GØ TØ 127
  124 A=A+.05
      GØ TØ 104
  127 FRP(K)=FRP(K)+.031415927*FØFA/APR
      N=I
  106 PHI=PHI+.031415927
  104 FRP(K)=FRP(K)*ALPHR*RP
  109 RP=RP+.05
      WRELA=(FRP(1)+FRP(21))/2.
      DØ 103 N=2,20
  103 WRELA=WRELA+FRP(N)
      WRELA=WRELA/31.415927
      PUNCH 1,WRELA
  110 ALPHR=ALPHR*SR
      STØP
      END
```

PART 2

```
C     PART 2, 4-LEVEL LASER EFFICIENCY AND GAIN PROGRAM (IBM 1620).
C     THIS PRØGRAM CALCULATES THE PRØDUCT ØF CAPTURE EFFICIENCY AND
C     RELAXATIØN EFFICIENCY VS. RADIUS TIMES DØPING DENSITY.
C     RUNNING TIME APPRØXIMATELY  25 MINUTES.
      DIMENSIØN H(101),SIGMA(101),F(101),W(101),WRELA(21)
    1 FØRMAT(E14.8)
    2 FØRMAT(/41HSWITCH 1 ØN TØ REPRESENT LAMP SPECTRUM BY)
    3 FØRMAT(34H BLACK BØDY RADIATIØN, PUSH START./)
    4 FØRMAT(/42HENTER BLACK BØDY TEMPERATURE IN DEGREES K.)
    5 FØRMAT(/47HENTER LIMITS ØF ABSØRPTIØN SPECTRUM IN MICRØNS.)
    6 FØRMAT(/41HENTER LASER ØUTPUT WAVELENGTH IN MICRØNS.)
    7 FØRMAT(/25HENTER QUANTUM EFFICIENCY.)
      PRINT 2
      PRINT 3
      PAUSE
      PRINT 7
      ACCEPT,E
      PRINT 5
      ACCEPT,WV1,WV2
      IF(WV1-WV2)101,101,102
  102 X=WV1
      WV1=WV2
      WV2=X
  101 PRINT 6
      ACCEPT,WV
      X=WV1
      IF(SENSE SWITCH 1)103,104
  103 PRINT 4
      ACCEPT,T
      DØ 105 I=1,101
      READ,SIGMA(I)
      H(I)=(X**(-5))/(EXP(14496.35/(T*X))-1.)
      X=X+.01
      IF(X-WV2)105,105,106
  105 CØNTINUE
  104 DØ 107 I=1,101
      READ,SIGMA(I),H(I)
      X=X+.01
      IF(X-WV2)107,107,106
  107 CØNTINUE
  106 SUM=(H(I)+H(1))/2.
      J=I-1
      DØ 108 K=2,J
  108 SUM=SUM+H(K)
      DØ 109 K=1,I
  109 H(K)=H(K)*(WV2-WV1)/(.01*SUM)
      DØ 110 K=1,21
  110 READ 1,WRELA(K)
      SR=SQRT(2.)
      RHØR=.001
      DØ 111 K=1,21
      DØ 112 L=1,I
  112 F(L)=RHØR*SIGMA(L)
      X=WV1
      DØ 120 L=1,I
      ALPHR=.01
      DØ 113 M=1,21
      IF(ALPHR-F(L))113,114,115
```

```
113 ALPHR=ALPHR*SR
114 W(L)=WRELA(M)*H(L)*X
    GØ TØ 120
115 IF(M-1)117,117,118
117 W(L)=0.
    GØ TØ 120
118 W(L)=WRELA(M)-(LØG(ALPHR/F(L))*(WRELA(M)-WRELA(M-1))/LØG(SR))
    W(L)=W(L)*H(L)*X
120 X=X+.01
    SUM=(W(I)+W(1))/2.
    DØ 119 L=2,J
119 SUM=SUM+W(L)
    ETA=.01*SUM*E/(WV*(WV2-WV1))
    PUNCH 1,ETA
111 RHØR=RHØR*SR
    STØP
    END
```

PART 3

```
C     PART 3, 4-LEVEL LASER EFFICIENCY AND GAIN PRØGRAM (IBM 1620).
C     THIS PRØGRAM CALCULATES THE TIME TØ THRESHØLD AND EFFICIENCY ØF A
C     LASER ØSCILLATØR AND THE TIME TØ THRESHØLD,MAXIMUM EFFICIENCY AND
C     MAXIMUM GAIN ØF A LASER AMPLIFIER.
C     RUNNING TIMES (APPRØXIMATE) - PRECØMPUTATIØN,  2 MINUTES.
C     ØSCILLATØR PRØBLEM,  3 MINUTES.  AMPLIFIER PRØBLEM,  2 MINUTES.
      DIMENSIØN ETA(21),GAMMA(32)
    1 FØRMAT(/34HENTER RØD RADIUS AND LENGTH IN CM.)
    2 FØRMAT(/33HENTER REFLECTIVITY ØF ØUTPUT END.)
    3 FØRMAT(/51HENTER LAMP ENERGY IN JØULES AND DECAY TIME CØNSTANT)
    4 FØRMAT(17H IN MICRØSECØNDS.)
    5 FØRMAT(/36HENTER DØPING DENSITY IN IØNS PER CC.)
    6 FØRMAT(/41HENTER ØPERATING TEMPERATURE IN DEGREES K.)
    7 FØRMAT(/45HSWITCH 1 ØN FØR AMPLIFIER, ØFF FØR ØSCILLATØR)
    8 FØRMAT(13H, PUSH START./)
    9 FØRMAT(/45HENTER ABSØRPTIVITY ØF CLØSED END, ØUTPUT END.)
   10 FØRMAT(//19HRHØ R ØUT ØF RANGE./)
   11 FØRMAT(//44HTHRESHØLD NØT REACHED, INCREASE LAMP ENERGY./)
   12 FØRMAT(/44HENTER ATTENUATIØN CØNSTANT ØF HØST MATERIAL.)
   13 FØRMAT(E14.8)
   14 FØRMAT(//19HTIME TØ THRESHØLD =,F5.0,14H MICRØSECØNDS,)
   15 FØRMAT(/12HEFFICIENCY =,F11.8,9H PERCENT.)
   16 FØRMAT(/17HMAX. EFFICIENCY =,F11.8,21H PERCENT, MAX. GAIN =,F12.7)
   17 FØRMAT(/46HENTER LAMP EFFICIENCY, GEØMETRICAL EFFICIENCY.)
   18 FØRMAT(/50HENTER FLUØRESCENT LINE WIDTH IN CYCLES PER SECØND.)
   19 FØRMAT(/51HENTER LIFETIME ØF METASTABLE STATE IN MICRØSECØNDS.)
   20 FØRMAT(/48HENTER ENERGY DIFFERENCE BETWEEN GRØUND STATE AND)
   21 FØRMAT(33H TERMINAL STATE IN RECIPRØCAL CM.)
   22 FØRMAT(/41HENTER LASER ØUTPUT WAVELENGTH IN MICRØNS.)
   23 FØRMAT(///)
      DØ 101 N=1,21
  101 READ 13,ETA(N)
      PRINT 20
      PRINT 21
      ACCEPT,DEL
      PRINT 22
      ACCEPT,WV
```

```
      PRINT 19
      ACCEPT,TAU
      PRINT 17
      ACCEPT,ETAL,ETAG
      A=-ETAL*ETAG*WV*TAU/19.86
      T=20.
      DØ 100 N=1,32
      GAMMA(N)=EXP(-T/TAU)
100   T=T+20.
      PRINT 18
      ACCEPT,DELF
      PRINT 6
      ACCEPT,TK
      PRINT 23
123   PRINT 12
      ACCEPT,ALPHA
105   PRINT 5
      ACCEPT,RHØ
      PØP=RHØ*EXP(-1.4496*DEL/TK)
      PRINT 1
      ACCEPT,R,RL
      RHØR=1.E-20*RHØ*R
      IF(RHØR-.001)108,109,109
109   IF(1.024-RHØR)108,104,104
108   PRINT 10
      GØ TØ 105
104   PRINT 7
      PRINT 8
      PAUSE
      SIGMA=3.74E-04*(WV**2)/(DELF*TAU)
      IF(SENSE SWITCH 1)114,115
114   PØP=PØP+ALPHA/SIGMA
      GØ TØ 116
115   PRINT 2
      ACCEPT,R2
      PRINT 9
      ACCEPT,A1,A2
      R1=1.-A1
      PØP=PØP+(2.*ALPHA*RL-LØG(R1*R2))/(2.*SIGMA*RL)
116   PRINT 3
      PRINT 4
      ACCEPT,E1,TAU1
      Z=.001*SQRT(2.)
      DØ 117 N=2,21
      IF(RHØR-Z)118,119,117
117   Z=Z*SQRT(2.)
119   E=ETA(N)
      GØ TØ 120
118   E=ETA(N)-LØG(Z/RHØR)*(ETA(N)-ETA(N-1))/LØG(SQRT(2.))
120   TAU2=TAU1/10.35
      DEL1=TAU1-TAU2
      DEL2=TAU1/(TAU-TAU1)
      DEL3=TAU2/(TAU-TAU2)
      DEL4=DEL2+DEL3
      T=20.
      DØ 121 N=1,32
      P3=A*(DEL2*EXP(-T/TAU1)+DEL3*EXP(-T/TAU2)-DEL4*GAMMA(N))/DEL1
      P3=1.E+20*P3*E1*E/(3.1415927*(R**2)*RL)
      IF(P3-PØP)121,122,122
```

```
121 T=T+20.
    PRINT 11
    GØ TØ 116
122 PRINT 14,T
    EFF=TAU1*DEL2*EXP(-T/TAU1)+TAU2*DEL3*EXP(-T/TAU2)
    EFF=-1.E+02*(EFF-TAU*DEL4*GAMMA(N))/DEL1*ETAL*ETAG*E
    IF(SENSE SWITCH 1)102,103
103 GNET=-LØG(R1*R2)/(2.*RL)
    DEN=GNET*(1.-R2)
    DEN=DEN+ALPHA*R2*(1.+R1*EXP(GNET*RL))*(EXP(GNET*RL)-1.)
    DEN=DEN+A1*R2*GNET*EXP(GNET*RL)
    EFF=EFF*GNET*(1.-R2-A2)/DEN
    PRINT 15,EFF
    GØ TØ 106
102 DØ 129 K=N,32
    F=P3
    P3=A*(DEL2*EXP(-T/TAU1)+DEL3*EXP(-T/TAU2)-DEL4*GAMMA(K))/DEL1
    P3=1.E+20*P3*E1*E/(3.1415927*(R**2)*RL)
    IF(P3-F)130,129,129
129 T=T+20.
    F=P3
130 G=EXP(RL*SIGMA*(F-PØP))
    EFF=EFF/(1.+ALPHA*RL)
    PRINT 16,EFF,G
106 PRINT 23
    PAUSE
    GØ TØ 123
    END
```

Appendix B

Suggested Topics for Future Research

1. The development of accurate experimental methods for the measurement of the physical properties of Laser materials.

2. The measurement and tabulation of the physical properties of Laser materials under various conditions.

3. An investigation of the efficiency of present pumping sources and Laser pumping geometries.

4. A detailed study of the spectral output of Laser pumping sources.

5. An experimental check of the results of the computer model developed in this paper.

6. An extension of the applicability of the computer model to elliptical pumping geometries, and to the continuous, pulsed reflector, and hair-trigger modes of operation.

Appendix C

Computer Printout for Sample Problem

```
ENTER ENERGY DIFFERENCE BETWEEN GRØUND STATE AND TERMINAL STATE IN RECIPRØCAL CM.
1930
ENTER LASER ØUTPUT WAVELENGTH IN MICRØNS.
1.062
ENTER LIFETIME ØF METASTABLE STATE IN MICRØSECØNDS.
480
ENTER LAMP EFFICIENCY, GEØMETRICAL EFFICIENCY.
.25 .72
ENTER FLUØRESCENT LINE WIDTH IN CYCLES PER SECØND.
1.19E13
ENTER ØPERATING TEMPERATURE IN DEGREES K.
300


ENTER ATTENUATIØN CØNSTANT ØF HØST MATERIAL.
.015
ENTER DØPING DENSITY IN IØNS PER CC.
7E19
ENTER RØD RADIUS AND LENGTH IN CM.
.3 25
SWITCH 1 ØN FØR AMPLIFIER, ØFF FØR ØSCILLATØR, PUSH START.

ENTER REFLECTIVITY ØF ØUTPUT END.
.5
ENTER ABSØRPTIVITY ØF CLØSED END, ØUTPUT END.
.002 .001
ENTER LAMP ENERGY IN JØULES AND DECAY TIME CØNSTANT IN MICRØSECØNDS.
750 300

TIME TØ THRESHØLD =  40. MICRØSECØNDS,
EFFICIENCY =  .28623733 PERCENT.


ENTER ATTENUATIØN CØNSTANT ØF HØST MATERIAL.
.015
ENTER DØPING DENSITY IN IØNS PER CC.
7E19
ENTER RØD RADIUS AND LENGTH IN CM.
.3 25
SWITCH 1 ØN FØR AMPLIFIER, ØFF FØR ØSCILLATØR, PUSH START.

ENTER REFLECTIVITY ØF ØUTPUT END.
.6
ENTER ABSØRPTIVITY ØF CLØSED END, ØUTPUT END.
.002 .0012
ENTER LAMP ENERGY IN JØULES AND DECAY TIME CØNSTANT IN MICRØSECØNDS.
750 300

TIME TØ THRESHØLD =  40. MICRØSECØNDS,
EFFICIENCY =  .24115724 PERCENT.


ENTER ATTENUATIØN CØNSTANT ØF HØST MATERIAL.
.015
ENTER DØPING DENSITY IN IØNS PER CC.
7E19
ENTER RØD RADIUS AND LENGTH IN CM.
```

```
.3 25
SWITCH 1 ØN FØR AMPLIFIER, ØFF FØR ØSCILLATØR, PUSH START.

ENTER REFLECTIVITY ØF ØUTPUT END.
.7
ENTER ABSØRPTIVITY ØF CLØSED END, ØUTPUT END.
.002 .0014
ENTER LAMP ENERGY IN JØULES AND DECAY TIME CØNSTANT IN MICRØSECØNDS.
750 300

TIME TØ THRESHØLD =  20. MICRØSECØNDS,
EFFICIENCY =  .19258235 PERCENT.


ENTER ATTENUATIØN CØNSTANT ØF HØST MATERIAL.
.015
ENTER DØPING DENSITY IN IØNS PER CC.
7E19
ENTER RØD RADIUS AND LENGTH IN CM.
.3 25
SWITCH 1 ØN FØR AMPLIFIER, ØFF FØR ØSCILLATØR, PUSH START.

ENTER REFLECTIVITY ØF ØUTPUT END.
.8
ENTER ABSØRPTIVITY ØF CLØSED END, ØUTPUT END.
.002 .0016
ENTER LAMP ENERGY IN JØULES AND DECAY TIME CØNSTANT IN MICRØSECØNDS.
750 300

TIME TØ THRESHØLD =  20. MICRØSECØNDS,
EFFICIENCY =  .13652181 PERCENT.


ENTER ATTENUATIØN CØNSTANT ØF HØST MATERIAL.
.015
ENTER DØPING DENSITY IN IØNS PER CC.
7E19
ENTER RØD RADIUS AND LENGTH IN CM.
.3 25
SWITCH 1 ØN FØR AMPLIFIER, ØFF FØR ØSCILLATØR, PUSH START.

ENTER REFLECTIVITY ØF ØUTPUT END.
.9
ENTER ABSØRPTIVITY ØF CLØSED END, ØUTPUT END.
.002 .0018
ENTER LAMP ENERGY IN JØULES AND DECAY TIME CØNSTANT IN MICRØSECØNDS.
750 300

TIME TØ THRESHØLD =  20. MICRØSECØNDS,
EFFICIENCY =  .07257654 PERCENT.


ENTER ATTENUATIØN CØNSTANT ØF HØST MATERIAL.
.015
ENTER DØPING DENSITY IN IØNS PER CC.
7E19
ENTER RØD RADIUS AND LENGTH IN CM.
```

```
.4 25
SWITCH 1 ØN FØR AMPLIFIER, ØFF FØR ØSCILLATØR, PUSH START.

ENTER REFLECTIVITY ØF ØUTPUT END.
.5
ENTER ABSØRPTIVITY ØF CLØSED END, ØUTPUT END.
.002 .001
ENTER LAMP ENERGY IN JØULES AND DECAY TIME CØNSTANT IN MICRØSECØNDS.
1000 400

TIME TØ THRESHØLD =  40. MICRØSECØNDS,
EFFICIENCY =  .33482683 PERCENT.


ENTER ATTENUATIØN CØNSTANT ØF HØST MATERIAL.
.015
ENTER DØPING DENSITY IN IØNS PER CC.
7E19
ENTER RØD RADIUS AND LENGTH IN CM.
.4 25
SWITCH 1 ØN FØR AMPLIFIER, ØFF FØR ØSCILLATØR, PUSH START.

ENTER REFLECTIVITY ØF ØUTPUT END.
.6
ENTER ABSØRPTIVITY ØF CLØSED END, ØUTPUT END.
.002 .0012
ENTER LAMP ENERGY IN JØULES AND DECAY TIME CØNSTANT IN MICRØSECØNDS.
1000 400

TIME TØ THRESHØLD =  40. MICRØSECØNDS,
EFFICIENCY =  .28209428 PERCENT.


ENTER ATTENUATIØN CØNSTANT ØF HØST MATERIAL.
.015
ENTER DØPING DENSITY IN IØNS PER CC.
7E19
ENTER RØD RADIUS AND LENGTH IN CM.
.4 25
SWITCH 1 ØN FØR AMPLIFIER, ØFF FØR ØSCILLATØR, PUSH START.

ENTER REFLECTIVITY ØF ØUTPUT END.
.7
ENTER ABSØRPTIVITY ØF CLØSED END, ØUTPUT END.
.002 .0014
ENTER LAMP ENERGY IN JØULES AND DECAY TIME CØNSTANT IN MICRØSECØNDS.
1000 400

TIME TØ THRESHØLD =  40. MICRØSECØNDS,
EFFICIENCY =  .22397586 PERCENT.


ENTER ATTENUATIØN CØNSTANT ØF HØST MATERIAL.
.015
ENTER DØPING DENSITY IN IØNS PER CC.
7E19
ENTER RØD RADIUS AND LENGTH IN CM.
```

```
.4 25
SWITCH 1 ØN FØR AMPLIFIER, ØFF FØR ØSCILLATØR, PUSH START.

ENTER REFLECTIVITY ØF ØUTPUT END.
.8
ENTER ABSØRPTIVITY ØF CLØSED END, ØUTPUT END.
.002 .0016
ENTER LAMP ENERGY IN JØULES AND DECAY TIME CØNSTANT IN MICRØSECØNDS.
1000 400

TIME TØ THRESHØLD =  40. MICRØSECØNDS,
EFFICIENCY =  .15877670 PERCENT.


ENTER ATTENUATIØN CØNSTANT ØF HØST MATERIAL.
.015
ENTER DØPING DENSITY IN IØNS PER CC.
7E19
ENTER RØD RADIUS AND LENGTH IN CM.
.4 25
SWITCH 1 ØN FØR AMPLIFIER, ØFF FØR ØSCILLATØR, PUSH START.

ENTER REFLECTIVITY ØF ØUTPUT END.
.9
ENTER ABSØRPTIVITY ØF CLØSED END, ØUTPUT END.
.002 .0018
ENTER LAMP ENERGY IN JØULES AND DECAY TIME CØNSTANT IN MICRØSECØNDS.
1000 400

TIME TØ THRESHØLD =  40. MICRØSECØNDS,
EFFICIENCY =  .08440750 PERCENT.


ENTER ATTENUATIØN CØNSTANT ØF HØST MATERIAL.
.015
ENTER DØPING DENSITY IN IØNS PER CC.
7E19
ENTER RØD RADIUS AND LENGTH IN CM.
.5 25
SWITCH 1 ØN FØR AMPLIFIER, ØFF FØR ØSCILLATØR, PUSH START.

ENTER REFLECTIVITY ØF ØUTPUT END.
.5
ENTER ABSØRPTIVITY ØF CLØSED END, ØUTPUT END.
.002 .001
ENTER LAMP ENERGY IN JØULES AND DECAY TIME CØNSTANT IN MICRØSECØNDS.
1250 500

TIME TØ THRESHØLD =  60. MICRØSECØNDS,
EFFICIENCY =  .37345961 PERCENT.


ENTER ATTENUATIØN CØNSTANT ØF HØST MATERIAL.
.015
ENTER DØPING DENSITY IN IØNS PER CC.
7E19
ENTER RØD RADIUS AND LENGTH IN CM.
```

```
.5 25
SWITCH 1 ØN FØR AMPLIFIER, ØFF FØR ØSCILLATØR, PUSH START.

ENTER REFLECTIVITY ØF ØUTPUT END.
.6
ENTER ABSØRPTIVITY ØF CLØSED END, ØUTPUT END.
.002 .0012
ENTER LAMP ENERGY IN JØULES AND DECAY TIME CØNSTANT IN MICRØSECØNDS.
1250 500

TIME TØ THRESHØLD =  60. MICRØSECØNDS,
EFFICIENCY =  .31464271 PERCENT.


ENTER ATTENUATIØN CØNSTANT ØF HØST MATERIAL.
.015
ENTER DØPING DENSITY IN IØNS PER CC.
7E19
ENTER RØD RADIUS AND LENGTH IN CM.
.5 25
SWITCH 1 ØN FØR AMPLIFIER, ØFF FØR ØSCILLATØR, PUSH START.

ENTER REFLECTIVITY ØF ØUTPUT END.
.7
ENTER ABSØRPTIVITY ØF CLØSED END, ØUTPUT END.
.002 .0014
ENTER LAMP ENERGY IN JØULES AND DECAY TIME CØNSTANT IN MICRØSECØNDS.
1250 500

TIME TØ THRESHØLD =  40. MICRØSECØNDS,
EFFICIENCY =  .25124405 PERCENT.


ENTER ATTENUATIØN CØNSTANT ØF HØST MATERIAL.
.015
ENTER DØPING DENSITY IN IØNS PER CC.
7E19
ENTER RØD RADIUS AND LENGTH IN CM.
.5 25
SWITCH 1 ØN FØR AMPLIFIER, ØFF FØR ØSCILLATØR, PUSH START.

ENTER REFLECTIVITY ØF ØUTPUT END.
.8
ENTER ABSØRPTIVITY ØF CLØSED END, ØUTPUT END.
.002 .0016
ENTER LAMP ENERGY IN JØULES AND DECAY TIME CØNSTANT IN MICRØSECØNDS.
1250 500

TIME TØ THRESHØLD =  40. MICRØSECØNDS,
EFFICIENCY =  .17810714 PERCENT.


ENTER ATTENUATIØN CØNSTANT ØF HØST MATERIAL.
.015
ENTER DØPING DENSITY IN IØNS PER CC.
7E19
ENTER RØD RADIUS AND LENGTH IN CM.
```

```
.5 25
SWITCH 1 ØN FØR AMPLIFIER, ØFF FØR ØSCILLATØR, PUSH START.

ENTER REFLECTIVITY ØF ØUTPUT END.
.9
ENTER ABSØRPTIVITY ØF CLØSED END, ØUTPUT END.
.002 .0018
ENTER LAMP ENERGY IN JØULES AND DECAY TIME CØNSTANT IN MICRØSECØNDS.
1250 500

TIME TØ THRESHØLD =  40. MICRØSECØNDS,
EFFICIENCY =  .09468377 PERCENT.


ENTER ATTENUATIØN CØNSTANT ØF HØST MATERIAL.
.015
ENTER DØPING DENSITY IN IØNS PER CC.
7E19
ENTER RØD RADIUS AND LENGTH IN CM.
.6 25
SWITCH 1 ØN FØR AMPLIFIER, ØFF FØR ØSCILLATØR, PUSH START.

ENTER REFLECTIVITY ØF ØUTPUT END.
.5
ENTER ABSØRPTIVITY ØF CLØSED END, ØUTPUT END.
.002 .001
ENTER LAMP ENERGY IN JØULES AND DECAY TIME CØNSTANT IN MICRØSECØNDS.
1500 600

TIME TØ THRESHØLD =  80. MICRØSECØNDS,
EFFICIENCY =  .40501937 PERCENT.


ENTER ATTENUATIØN CØNSTANT ØF HØST MATERIAL.
.015
ENTER DØPING DENSITY IN IØNS PER CC.
7E19
ENTER RØD RADIUS AND LENGTH IN CM.
.6 25
SWITCH 1 ØN FØR AMPLIFIER, ØFF FØR ØSCILLATØR, PUSH START.

ENTER REFLECTIVITY ØF ØUTPUT END.
.6
ENTER ABSØRPTIVITY ØF CLØSED END, ØUTPUT END.
.002 .0012
ENTER LAMP ENERGY IN JØULES AND DECAY TIME CØNSTANT IN MICRØSECØNDS.
1500 600

TIME TØ THRESHØLD =  80. MICRØSECØNDS,
EFFICIENCY =  .34123207 PERCENT.


ENTER ATTENUATIØN CØNSTANT ØF HØST MATERIAL.
.015
ENTER DØPING DENSITY IN IØNS PER CC.
7E19
ENTER RØD RADIUS AND LENGTH IN CM.
```

```
.6 25
SWITCH 1 ØN FØR AMPLIFIER, ØFF FØR ØSCILLATØR, PUSH START.

ENTER REFLECTIVITY ØF ØUTPUT END.
.7
ENTER ABSØRPTIVITY ØF CLØSED END, ØUTPUT END.
.002 .0014
ENTER LAMP ENERGY IN JØULES AND DECAY TIME CØNSTANT IN MICRØSECØNDS.
1500 600

TIME TØ THRESHØLD =  60. MICRØSECØNDS,
EFFICIENCY =  .27264684 PERCENT.


ENTER ATTENUATIØN CØNSTANT ØF HØST MATERIAL.
.015
ENTER DØPING DENSITY IN IØNS PER CC.
7E19
ENTER RØD RADIUS AND LENGTH IN CM.
.6 25
SWITCH 1 ØN FØR AMPLIFIER, ØFF FØR ØSCILLATØR, PUSH START.

ENTER REFLECTIVITY ØF ØUTPUT END.
.8
ENTER ABSØRPTIVITY ØF CLØSED END, ØUTPUT END.
.002 .0016
ENTER LAMP ENERGY IN JØULES AND DECAY TIME CØNSTANT IN MICRØSECØNDS.
1500 600

TIME TØ THRESHØLD =  60. MICRØSECØNDS,
EFFICIENCY =  .19327961 PERCENT.


ENTER ATTENUATIØN CØNSTANT ØF HØST MATERIAL.
.015
ENTER DØPING DENSITY IN IØNS PER CC.
7E19
ENTER RØD RADIUS AND LENGTH IN CM.
.6 25
SWITCH 1 ØN FØR AMPLIFIER, ØFF FØR ØSCILLATØR, PUSH START.

ENTER REFLECTIVITY ØF ØUTPUT END.
.9
ENTER ABSØRPTIVITY ØF CLØSED END, ØUTPUT END.
.002 .0018
ENTER LAMP ENERGY IN JØULES AND DECAY TIME CØNSTANT IN MICRØSECØNDS.
1500 600

TIME TØ THRESHØLD =  40. MICRØSECØNDS,
EFFICIENCY =  .10325479 PERCENT.


ENTER ATTENUATIØN CØNSTANT ØF HØST MATERIAL.
.015
ENTER DØPING DENSITY IN IØNS PER CC.
7E19
ENTER RØD RADIUS AND LENGTH IN CM.
```

```
.7 25
SWITCH 1 ON FOR AMPLIFIER, OFF FOR OSCILLATOR, PUSH START.

ENTER REFLECTIVITY OF OUTPUT END.
.5
ENTER ABSORPTIVITY OF CLOSED END, OUTPUT END.
.002 .001
ENTER LAMP ENERGY IN JOULES AND DECAY TIME CONSTANT IN MICROSECONDS.
1750 700

TIME TO THRESHOLD = 120. MICROSECONDS,
EFFICIENCY =  .42796621 PERCENT.


ENTER ATTENUATION CONSTANT OF HOST MATERIAL.
.015
ENTER DOPING DENSITY IN IONS PER CC.
7E19
ENTER ROD RADIUS AND LENGTH IN CM.
.7 25
SWITCH 1 ON FOR AMPLIFIER, OFF FOR OSCILLATOR, PUSH START.

ENTER REFLECTIVITY OF OUTPUT END.
.6
ENTER ABSORPTIVITY OF CLOSED END, OUTPUT END.
.002 .0012
ENTER LAMP ENERGY IN JOULES AND DECAY TIME CONSTANT IN MICROSECONDS.
1750 700

TIME TO THRESHOLD = 100. MICROSECONDS,
EFFICIENCY =  .36333719 PERCENT.


ENTER ATTENUATION CONSTANT OF HOST MATERIAL.
.015
ENTER DOPING DENSITY IN IONS PER CC.
7E19
ENTER ROD RADIUS AND LENGTH IN CM.
.7 25
SWITCH 1 ON FOR AMPLIFIER, OFF FOR OSCILLATOR, PUSH START.

ENTER REFLECTIVITY OF OUTPUT END.
.7
ENTER ABSORPTIVITY OF CLOSED END, OUTPUT END.
.002 .0014
ENTER LAMP ENERGY IN JOULES AND DECAY TIME CONSTANT IN MICROSECONDS.
1750 700

TIME TO THRESHOLD =  80. MICROSECONDS,
EFFICIENCY =  .29041761 PERCENT.


ENTER ATTENUATION CONSTANT OF HOST MATERIAL.
.015
ENTER DOPING DENSITY IN IONS PER CC.
7E19
ENTER ROD RADIUS AND LENGTH IN CM.
```

```
.7 25
SWITCH 1 ØN FØR AMPLIFIER, ØFF FØR ØSCILLATØR, PUSH START.

ENTER REFLECTIVITY ØF ØUTPUT END.
.8
ENTER ABSØRPTIVITY ØF CLØSED END, ØUTPUT END.
.002 .0016
ENTER LAMP ENERGY IN JØULES AND DECAY TIME CØNSTANT IN MICRØSECØNDS.
1750 700

TIME TØ THRESHØLD =  80. MICRØSECØNDS,
EFFICIENCY =  .20587731 PERCENT.


ENTER ATTENUATIØN CØNSTANT ØF HØST MATERIAL.
.015
ENTER DØPING DENSITY IN IØNS PER CC.
7E19
ENTER RØD RADIUS AND LENGTH IN CM.
.7 25
SWITCH 1 ØN FØR AMPLIFIER, ØFF FØR ØSCILLATØR, PUSH START.

ENTER REFLECTIVITY ØF ØUTPUT END.
.9
ENTER ABSØRPTIVITY ØF CLØSED END, ØUTPUT END.
.002 .0018
ENTER LAMP ENERGY IN JØULES AND DECAY TIME CØNSTANT IN MICRØSECØNDS.
1750 700

TIME TØ THRESHØLD =  60. MICRØSECØNDS,
EFFICIENCY =  .11006010 PERCENT.


ENTER ATTENUATIØN CØNSTANT ØF HØST MATERIAL.
.015
ENTER DØPING DENSITY IN IØNS PER CC.
7E19
ENTER RØD RADIUS AND LENGTH IN CM.
.8 25
SWITCH 1 ØN FØR AMPLIFIER, ØFF FØR ØSCILLATØR, PUSH START.

ENTER REFLECTIVITY ØF ØUTPUT END.
.5
ENTER ABSØRPTIVITY ØF CLØSED END, ØUTPUT END.
.002 .001
ENTER LAMP ENERGY IN JØULES AND DECAY TIME CØNSTANT IN MICRØSECØNDS.
2000 800

TIME TØ THRESHØLD = 160. MICRØSECØNDS,
EFFICIENCY =  .44654105 PERCENT.


ENTER ATTENUATIØN CØNSTANT ØF HØST MATERIAL.
.015
ENTER DØPING DENSITY IN IØNS PER CC.
7E19
ENTER RØD RADIUS AND LENGTH IN CM.
```

```
.8 25
SWITCH 1 ØN FØR AMPLIFIER, ØFF FØR ØSCILLATØR, PUSH START.

ENTER REFLECTIVITY ØF ØUTPUT END.
.6
ENTER ABSØRPTIVITY ØF CLØSED END, ØUTPUT END.
.002 .0012
ENTER LAMP ENERGY IN JØULES AND DECAY TIME CØNSTANT IN MICRØSECØNDS.
2000 800

TIME TØ THRESHØLD = 120. MICRØSECØNDS,
EFFICIENCY =  .38233702 PERCENT.


ENTER ATTENUATIØN CØNSTANT ØF HØST MATERIAL.
.015
ENTER DØPING DENSITY IN IØNS PER CC.
7E19
ENTER RØD RADIUS AND LENGTH IN CM.
.8 25
SWITCH 1 ØN FØR AMPLIFIER, ØFF FØR ØSCILLATØR, PUSH START.

ENTER REFLECTIVITY ØF ØUTPUT END.
.7
ENTER ABSØRPTIVITY ØF CLØSED END, ØUTPUT END.
.002 .0014
ENTER LAMP ENERGY IN JØULES AND DECAY TIME CØNSTANT IN MICRØSECØNDS.
2000 800

TIME TØ THRESHØLD = 100. MICRØSECØNDS,
EFFICIENCY =  .30567409 PERCENT.


ENTER ATTENUATIØN CØNSTANT ØF HØST MATERIAL.
.015
ENTER DØPING DENSITY IN IØNS PER CC.
7E19
ENTER RØD RADIUS AND LENGTH IN CM.
.8 25
SWITCH 1 ØN FØR AMPLIFIER, ØFF FØR ØSCILLATØR, PUSH START.

ENTER REFLECTIVITY ØF ØUTPUT END.
.8
ENTER ABSØRPTIVITY ØF CLØSED END, ØUTPUT END.
.002 .0016
ENTER LAMP ENERGY IN JØULES AND DECAY TIME CØNSTANT IN MICRØSECØNDS.
2000 800

TIME TØ THRESHØLD =  80. MICRØSECØNDS,
EFFICIENCY =  .21800274 PERCENT.


ENTER ATTENUATIØN CØNSTANT ØF HØST MATERIAL.
.015
ENTER DØPING DENSITY IN IØNS PER CC.
7E19
ENTER RØD RADIUS AND LENGTH IN CM.
```

```
.8 25
SWITCH 1 ØN FØR AMPLIFIER, ØFF FØR ØSCILLATØR, PUSH START.

ENTER REFLECTIVITY ØF ØUTPUT END.
.9
ENTER ABSØRPTIVITY ØF CLØSED END, ØUTPUT END.
.002 .0018
ENTER LAMP ENERGY IN JØULES AND DECAY TIME CØNSTANT IN MICRØSECØNDS.
2000 800

TIME TØ THRESHØLD =  80. MICRØSECØNDS,
EFFICIENCY =  .11589273 PERCENT.


ENTER ATTENUATIØN CØNSTANT ØF HØST MATERIAL.
.015
ENTER DØPING DENSITY IN IØNS PER CC.
7E19
ENTER RØD RADIUS AND LENGTH IN CM.
.9 25
SWITCH 1 ØN FØR AMPLIFIER, ØFF FØR ØSCILLATØR, PUSH START.

ENTER REFLECTIVITY ØF ØUTPUT END.
.5
ENTER ABSØRPTIVITY ØF CLØSED END, ØUTPUT END.
.002 .001
ENTER LAMP ENERGY IN JØULES AND DECAY TIME CØNSTANT IN MICRØSECØNDS.
2250 900

TIME TØ THRESHØLD = 200. MICRØSECØNDS,
EFFICIENCY =  .46175347 PERCENT.


ENTER ATTÉNUATIØN CØNSTANT ØF HØST MATERIAL.
.015
ENTER DØPING DENSITY IN IØNS PER CC.
7E19
ENTER RØD RADIUS AND LENGTH IN CM.
.9 25
SWITCH 1 ØN FØR AMPLIFIER, ØFF FØR ØSCILLATØR, PUSH START.

ENTER REFLECTIVITY ØF ØUTPUT END.
.6
ENTER ABSØRPTIVITY ØF CLØSED END, ØUTPUT END.
.002 .0012
ENTER LAMP ENERGY IN JØULES AND DECAY TIME CØNSTANT IN MICRØSECØNDS.
2250 900

TIME TØ THRESHØLD = 160. MICRØSECØNDS,
EFFICIENCY =  .39578184 PERCENT.


ENTER ATTENUATIØN CØNSTANT ØF HØST MATERIAL.
.015
ENTER DØPING DENSITY IN IØNS PER CC.
7E19
ENTER RØD RADIUS AND LENGTH IN CM.
```

```
.9 25
SWITCH 1 ØN FØR AMPLIFIER, ØFF FØR ØSCILLATØR, PUSH START.

ENTER REFLECTIVITY ØF ØUTPUT END.
.7
ENTER ABSØRPTIVITY ØF CLØSED END, ØUTPUT END.
.002 .0014
ENTER LAMP ENERGY IN JØULES AND DECAY TIME CØNSTANT IN MICRØSECØNDS.
2250 900

TIME TØ THRESHØLD = 140. MICRØSECØNDS,
EFFICIENCY =  .31667714 PERCENT.


ENTER ATTENUATIØN CØNSTANT ØF HØST MATERIAL.
.015
ENTER DØPING DENSITY IN IØNS PER CC.
7E19
ENTER RØD RADIUS AND LENGTH IN CM.
.9 25
SWITCH 1 ØN FØR AMPLIFIER, ØFF FØR ØSCILLATØR, PUSH START.

ENTER REFLECTIVITY ØF ØUTPUT END.
.8
ENTER ABSØRPTIVITY ØF CLØSED END, ØUTPUT END.
.002 .0016
ENTER LAMP ENERGY IN JØULES AND DECAY TIME CØNSTANT IN MICRØSECØNDS.
2250 900

TIME TØ THRESHØLD = 120. MICRØSECØNDS,
EFFICIENCY =  .22608244 PERCENT.


ENTER ATTENUATIØN CØNSTANT ØF HØST MATERIAL.
.015
ENTER DØPING DENSITY IN IØNS PER CC.
7E19
ENTER RØD RADIUS AND LENGTH IN CM.
.9 25
SWITCH 1 ØN FØR AMPLIFIER, ØFF FØR ØSCILLATØR, PUSH START.

ENTER REFLECTIVITY ØF ØUTPUT END.
.9
ENTER ABSØRPTIVITY ØF CLØSED END, ØUTPUT END.
.002 .0018
ENTER LAMP ENERGY IN JØULES AND DECAY TIME CØNSTANT IN MICRØSECØNDS.
2250 900

TIME TØ THRESHØLD = 100. MICRØSECØNDS,
EFFICIENCY =  .12094952 PERCENT.


ENTER ATTENUATIØN CØNSTANT ØF HØST MATERIAL.
.015
ENTER DØPING DENSITY IN IØNS PER CC.
7E19
ENTER RØD RADIUS AND LENGTH IN CM.
```

```
1 25
SWITCH 1 ØN FØR AMPLIFIER, ØFF FØR ØSCILLATØR, PUSH START.

ENTER REFLECTIVITY ØF ØUTPUT END.
.5
ENTER ABSØRPTIVITY ØF CLØSED END, ØUTPUT END.
.002 .001
ENTER LAMP ENERGY IN JØULES AND DECAY TIME CØNSTANT IN MICRØSECØNDS.
2500 1000

TIME TØ THRESHØLD = 280. MICRØSECØNDS,
EFFICIENCY =  .46515682 PERCENT.


ENTER ATTENUATIØN CØNSTANT ØF HØST MATERIAL.
.015
ENTER DØPING DENSITY IN IØNS PER CC.
7E19
ENTER RØD RADIUS AND LENGTH IN CM.
1 25
SWITCH 1 ØN FØR AMPLIFIER, ØFF FØR ØSCILLATØR, PUSH START.

ENTER REFLECTIVITY ØF ØUTPUT END.
.6
ENTER ABSØRPTIVITY ØF CLØSED END, ØUTPUT END.
.002 .0012
ENTER LAMP ENERGY IN JØULES AND DECAY TIME CØNSTANT IN MICRØSECØNDS.
2500 1000

TIME TØ THRESHØLD = 200. MICRØSECØNDS,
EFFICIENCY =  .40678021 PERCENT.


ENTER ATTENUATIØN CØNSTANT ØF HØST MATERIAL.
.015
ENTER DØPING DENSITY IN IØNS PER CC.
7E19
ENTER RØD RADIUS AND LENGTH IN CM.
1 25
SWITCH 1 ØN FØR AMPLIFIER, ØFF FØR ØSCILLATØR, PUSH START.

ENTER REFLECTIVITY ØF ØUTPUT END.
.7
ENTER ABSØRPTIVITY ØF CLØSED END, ØUTPUT END.
.002 .0014
ENTER LAMP ENERGY IN JØULES AND DECAY TIME CØNSTANT IN MICRØSECØNDS.
2500 1000

TIME TØ THRESHØLD = 160. MICRØSECØNDS,
EFFICIENCY =  .32815479 PERCENT.


ENTER ATTENUATIØN CØNSTANT ØF HØST MATERIAL.
.015
ENTER DØPING DENSITY IN IØNS PER CC.
7E19
ENTER RØD RADIUS AND LENGTH IN CM.
```

```
1 25
SWITCH 1 ØN FØR AMPLIFIER, ØFF FØR ØSCILLATØR, PUSH START.

ENTER REFLECTIVITY ØF ØUTPUT END.
.8
ENTER ABSØRPTIVITY ØF CLØSED END, ØUTPUT END.
.002 .0016
ENTER LAMP ENERGY IN JØULES AND DECAY TIME CØNSTANT IN MICRØSECØNDS.
2500 1000

TIME TØ THRESHØLD = 140. MICRØSECØNDS,
EFFICIENCY =  .23429432 PERCENT.


ENTER ATTENUATIØN CØNSTANT ØF HØST MATERIAL.
.015
ENTER DØPING DENSITY IN IØNS PER CC.
7E19
ENTER RØD RADIUS AND LENGTH IN CM.
1 25
SWITCH 1 ØN FØR AMPLIFIER, ØFF FØR ØSCILLATØR, PUSH START.

ENTER REFLECTIVITY ØF ØUTPUT END.
.9
ENTER ABSØRPTIVITY ØF CLØSED END, ØUTPUT END.
.002 .0018
ENTER LAMP ENERGY IN JØULES AND DECAY TIME CØNSTANT IN MICRØSECØNDS.
2500 1000

TIME TØ THRESHØLD = 120. MICRØSECØNDS,
EFFICIENCY =  .12536659 PERCENT.


ENTER ATTENUATIØN CØNSTANT ØF HØST MATERIAL.
.015
ENTER DØPING DENSITY IN IØNS PER CC.
7E19
ENTER RØD RADIUS AND LENGTH IN CM.
1.1 25
SWITCH 1 ØN FØR AMPLIFIER, ØFF FØR ØSCILLATØR, PUSH START.

ENTER REFLECTIVITY ØF ØUTPUT END.
.5
ENTER ABSØRPTIVITY ØF CLØSED END, ØUTPUT END.
.002 .001
ENTER LAMP ENERGY IN JØULES AND DECAY TIME CØNSTANT IN MICRØSECØNDS.
2750 1100

TIME TØ THRESHØLD = 440. MICRØSECØNDS,
EFFICIENCY =  .44557654 PERCENT.


ENTER ATTENUATIØN CØNSTANT ØF HØST MATERIAL.
.015
ENTER DØPING DENSITY IN IØNS PER CC.
7E19
ENTER RØD RADIUS AND LENGTH IN CM.
```

```
1.1 25
SWITCH 1 ON FOR AMPLIFIER, ØFF FØR ØSCILLATØR, PUSH START.

ENTER REFLECTIVITY ØF ØUTPUT END.
.6
ENTER ABSØRPTIVITY ØF CLØSED END, ØUTPUT END.
.002 .0012
ENTER LAMP ENERGY IN JØULES AND DECAY TIME CØNSTANT IN MICRØSECØNDS.
2750 1100

TIME TØ THRESHØLD = 280. MICRØSECØNDS,
EFFICIENCY =  .40847202 PERCENT.


ENTER ATTENUATIØN CØNSTANT ØF HØST MATERIAL.
.015
ENTER DØPING DENSITY IN IØNS PER CC.
7E19
ENTER RØD RADIUS AND LENGTH IN CM.
1.1 25
SWITCH 1 ØN FØR AMPLIFIER, ØFF FØR ØSCILLATØR, PUSH START.

ENTER REFLECTIVITY ØF ØUTPUT END.
.7
ENTER ABSØRPTIVITY ØF CLØSED END, ØUTPUT END.
.002 .0014
ENTER LAMP ENERGY IN JØULES AND DECAY TIME CØNSTANT IN MICRØSECØNDS.
2750 1100

TIME TØ THRESHØLD = 200. MICRØSECØNDS,
EFFICIENCY =  .33578956 PERCENT.


ENTER ATTENUATIØN CØNSTANT ØF HØST MATERIAL.
.015
ENTER DØPING DENSITY IN IØNS PER CC.
7E19
ENTER RØD RADIUS AND LENGTH IN CM.
1.1 25
SWITCH 1 ØN FØR AMPLIFIER, ØFF FØR ØSCILLATØR, PUSH START.

ENTER REFLECTIVITY ØF ØUTPUT END.
.8
ENTER ABSØRPTIVITY ØF CLØSED END, ØUTPUT END.
.002 .0016
ENTER LAMP ENERGY IN JØULES AND DECAY TIME CØNSTANT IN MICRØSECØNDS.
2750 1100

TIME TØ THRESHØLD = 160. MICRØSECØNDS,
EFFICIENCY =  .24159533 PERCENT.


ENTER ATTENUATIØN CØNSTANT ØF HØST MATERIAL.
.015
ENTER DØPING DENSITY IN IØNS PER CC.
7E19
ENTER RØD RADIUS AND LENGTH IN CM.
```

```
1.1 25
SWITCH 1 ØN FØR AMPLIFIER, ØFF FØR ØSCILLATØR, PUSH START.

ENTER REFLECTIVITY ØF ØUTPUT END.
.9
ENTER ABSØRPTIVITY ØF CLØSED END, ØUTPUT END.
.002 .0018
ENTER LAMP ENERGY IN JØULES AND DECAY TIME CØNSTANT IN MICRØSECØNDS.
2750 1100

TIME TØ THRESHØLD = 140. MICRØSECØNDS,
EFFICIENCY =  .12928915 PERCENT.


ENTER ATTENUATIØN CØNSTANT ØF HØST MATERIAL.
.015
ENTER DØPING DENSITY IN IØNS PER CC.
7E19
ENTER RØD RADIUS AND LENGTH IN CM.
1.2 25
SWITCH 1 ØN FØR AMPLIFIER, ØFF FØR ØSCILLATØR, PUSH START.

ENTER REFLECTIVITY ØF ØUTPUT END.
.5
ENTER ABSØRPTIVITY ØF CLØSED END, ØUTPUT END.
.002 .001
ENTER LAMP ENERGY IN JØULES AND DECAY TIME CØNSTANT IN MICRØSECØNDS.
3000 1200

THRESHØLD NØT REACHED, INCREASE LAMP ENERGY.

ENTER LAMP ENERGY IN JØULES AND DECAY TIME CØNSTANT IN MICRØSECØNDS.
1E6 1000

TIME TØ THRESHØLD =  20. MICRØSECØNDS,
EFFICIENCY =  .54229826 PERCENT.


ENTER ATTENUATIØN CØNSTANT ØF HØST MATERIAL.
.015
ENTER DØPING DENSITY IN IØNS PER CC.
7E19
ENTER RØD RADIUS AND LENGTH IN CM.
1.2 25
SWITCH 1 ØN FØR AMPLIFIER, ØFF FØR ØSCILLATØR, PUSH START.

ENTER REFLECTIVITY ØF ØUTPUT END.
.6
ENTER ABSØRPTIVITY ØF CLØSED END, ØUTPUT END.
.002 .0012
ENTER LAMP ENERGY IN JØULES AND DECAY TIME CØNSTANT IN MICRØSECØNDS.
3000 1200

TIME TØ THRESHØLD = 400. MICRØSECØNDS,
EFFICIENCY =  .39978959 PERCENT.
```

```
ENTER ATTENUATIØN CØNSTANT ØF HØST MATERIAL.
.015
ENTER DØPING DENSITY IN IØNS PER CC.
7E19
ENTER RØD RADIUS AND LENGTH IN CM.
1.2 25
SWITCH 1 ØN FØR AMPLIFIER, ØFF FØR ØSCILLATØR, PUSH START.

ENTER REFLECTIVITY ØF ØUTPUT END.
.7
ENTER ABSØRPTIVITY ØF CLØSED END, ØUTPUT END.
.002 .0014
ENTER LAMP ENERGY IN JØULES AND DECAY TIME CØNSTANT IN MICRØSECØNDS.
3000 1200

TIME TØ THRESHØLD = 260. MICRØSECØNDS,
EFFICIENCY =  .33931019 PERCENT.


ENTER ATTENUATIØN CØNSTANT ØF HØST MATERIAL.
.015
ENTER DØPING DENSITY IN IØNS PER CC.
7E19
ENTER RØD RADIUS AND LENGTH IN CM.
1.2 25
SWITCH 1 ØN FØR AMPLIFIER, ØFF FØR ØSCILLATØR, PUSH START.

ENTER REFLECTIVITY ØF ØUTPUT END.
.8
ENTER ABSØRPTIVITY ØF CLØSED END, ØUTPUT END.
.002 .0016
ENTER LAMP ENERGY IN JØULES AND DECAY TIME CØNSTANT IN MICRØSECØNDS.
3000 1200

TIME TØ THRESHØLD = 200. MICRØSECØNDS,
EFFICIENCY =  .24636118 PERCENT.


ENTER ATTENUATIØN CØNSTANT ØF HØST MATERIAL.
.015
ENTER DØPING DENSITY IN IØNS PER CC.
7E19
ENTER RØD RADIUS AND LENGTH IN CM.
1.2 25
SWITCH 1 ØN FØR AMPLIFIER, ØFF FØR ØSCILLATØR, PUSH START.

ENTER REFLECTIVITY ØF ØUTPUT END.
.9
ENTER ABSØRPTIVITY ØF CLØSED END, ØUTPUT END.
.002 .0018
ENTER LAMP ENERGY IN JØULES AND DECAY TIME CØNSTANT IN MICRØSECØNDS.
3000 1200

TIME TØ THRESHØLD = 160. MICRØSECØNDS,
EFFICIENCY =  .13279791 PERCENT.
```

Vita

Tom Gordon Purnhagen was born on 20 April 1934 in Columbus, Ohio, the son of John Frederick Purnhagen and Florence Esther Purnhagen. He graduated from Linden McKinley High School in Columbus in 1951, and received the Bachelor of Electrical Engineering degree from Ohio State University in June 1956. During his last two years at Ohio State, he worked as a laboratory assistant in the Electron Tube Laboratory of the Ohio State University Research Foundation. Upon graduation, he was commissioned a Second Lieutenant in the United States Air Force Reserve. He received his commission in the Regular Air Force in March 1958. Prior to his assignment to the Air Force Institute of Technology in June 1961, he served as Communications Officer and Civil Engineering Officer at several Air Defense Command radar squadrons in the United States and Canada. Captain Purnhagen is a member of Eta Kappa Nu and Phi Eta Sigma honorary societies.

Permanent Address: 2643 Jordan Road
Columbus 24, Ohio

This thesis was typed by the author.